# ANALITHICAL dETERMINATIONS REGARDING THE REPLACEMENT OF THE HELICAL RELIEF GRINDING WITH THE CONICAL RELIEF GRINDING OF CUTTING GEARS FOR BEVEL GEAR IN CIRCULAR ARC

**Horia TÂRZIU**

"Henri Coandă" Air Force Academy

**Abstract:** *Is known the percentage of the conical bevel gears with curved toothing used in mechanical transmissions from the cinematic chains of motor vehicles and armoured cars. The technology of the tools for conical curved toothing is expensive on the whole and, particularly, the relief grinding of cutting gears is expensive. The paper presents analytically an original method, simpler and cheaper, for generating the side surfaces for the blades of these face-mills (with bilateral cutting), by replacing the optimal surface (a helical one) with a straight conical circular surface that can be easily generated on universal circular reshaping tools. It is demonstrated, through calculus, that this substitution does not introduce geometrical changes greater than the regular imposed tolerance for this kind of relief grinding.*

**Key words:** *relief grinding, cutting gears, face-mills, directory curve.*

## 1. THE EQUATION OF THE SIDE SURFACES WITH HELICAL CURVE DIRECTORY

We consider the easy case, when the cutter is fixed and all the movements are done by a tool (rough stone). We choose two reference-systems: a fixed one Oxyz and a mobile one, connected with the tool $O_1x_1y_1z_1$. If BC, the considered generator has a translation movement following Ox (the axis of the gear cutter) and not Oy, then the surface directory turnes into a helical curve and the surface thus generated tures into a helical surface.

In the inside blade's case, to determine the equation of the side surface, the reference systems are chosen as in picture 1.

At the beginning the two reference systems are the same. The generator (the rough stone) is inclined with the angle of the inside profile $\alpha_I$ and it is considered that connected with the mobile reference system, executes all the movements: rotation around Ox axis and translation along this axis. The translation movement is towards the negative direction of the Ox axis, and that's why in the transfer matrix, the axial movement parameter has the "-" sign.

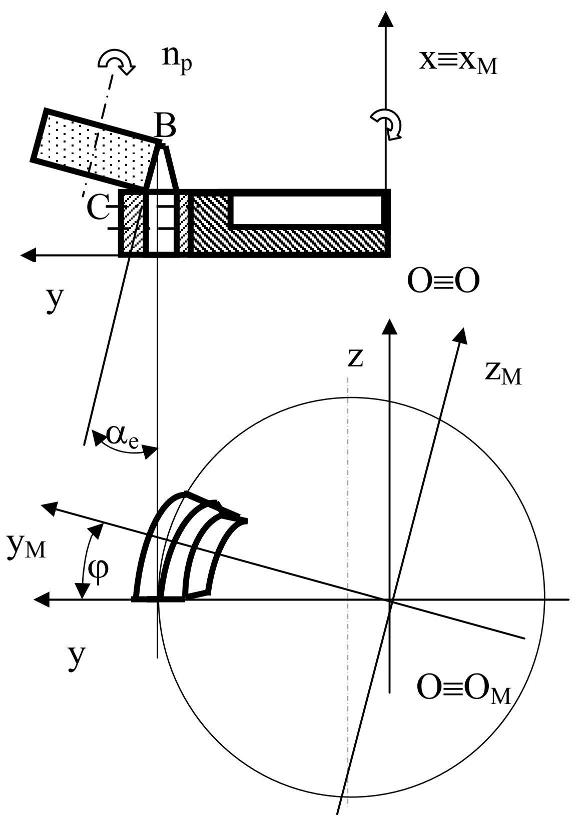


Fig. 1 Fix and mobil references system atached

We obtain the parametrical equations of the side surface for an inside cutter. These are:

$$\left.\begin{aligned} x &= \lambda \cos \alpha_i - \frac{p_e}{2\pi}\varphi \\ y &= \lambda \sin \alpha_i \cos \varphi \\ z &= \lambda \sin \alpha_i \sin \varphi \end{aligned}\right\} \quad (1)$$

The implicit form of the (1) equation is the following:

$$x = \operatorname{ctg}\alpha_i \sqrt{y^2 + z^2} - \frac{p_e}{2\pi}\operatorname{arctg}\frac{z}{y} \quad (2)$$

In order to calculate the angle and to study the variation of this angle in different perpendicular planes with the rotation axis of the gear cutter is more convenient the following form, in cylindrical co-ordinates, of the 1 equation, X being a parameter:

$$\rho(x_i, \varphi) = \operatorname{tg}\alpha_i\left(x_i + \frac{p_e\varphi}{2\pi}\right) \quad (3)$$

The angle of an inside blade on a side surface curve correlated with the $x_i$ plane, varies with φ, according to the formula:

$$\operatorname{tg}\alpha = \pm\frac{p_e}{p_e\varphi + 2\pi x_i} \quad (4)$$

We can analogous demonstrate that the parametrical equations of the surfaces in this way relief ground, are for an exterior blade:

$$\left.\begin{aligned} x &= \lambda \cos \alpha_e + \frac{p_e}{2\pi}\varphi \\ y &= \lambda \sin \alpha_e \cos \varphi \\ z &= \lambda \sin \alpha_e \sin \varphi \end{aligned}\right\} \quad (5)$$

By eliminating the λ and φ parameters from this formula the implicit form of the equation can be obtained. In cylindrical co-ordinates, the equation is:

$$\rho(\varphi) = \operatorname{tg}\alpha_e\left(x_i - \frac{p_e\varphi}{2\pi}\right) \quad (6)$$

It may be observed that the curves from different planes $x_I$ are equidistant having in between the following distance:

$$\Delta(\rho) = \operatorname{tg}\alpha_e(x_i - x_j) \quad (7)$$

Which does not depend on φ angle, thus showing that the profile of this surfaces is constant. Or, in a plane of φ = constant equation, the surfaces profile is given by the angle in between Ox axis and the equation's curve

$$\rho(x) = \operatorname{tg}\alpha_e\left(x_i - \frac{p_e\varphi}{2\pi}\right) \quad (8)$$

more precisely the angular coefficient of this curve, which does not depend on φ and is tg $\alpha_e$.

The angle of this surfaces varies and the variation can de studied with the formula:

$$\alpha = \pm\operatorname{arcctg}\left(\varphi - \frac{2\pi x_i}{p_e}\right) \quad (9)$$

The relief grinding of the side surface with this method has the following advantages: the maintenance of the tool's profile and the possibility of cinematic obtain of the directory curve.

### 1.1. THE APPROXIMATE OF THE HELICOIDAL SIDE SURFACES WITH A STRAIGHT CYLINDER CONICAL SURFACES

In a reference system whose Ox axis is the axis of the gear cutter, the side surfaces equation of an exterior blade, in cylindrical co-ordinates, is given by the following formula:

$$\rho(x, \varphi) = \operatorname{tg}\alpha_e\left(x - \frac{p_e\varphi}{2\pi}\right) \quad (10)$$

If this surfaces is intersected by the $x = x_I$ plane a arhimedic spiral is obtained whose equation in polar co-ordinates is:

$$\rho(\varphi) = \operatorname{tg}\alpha_e\left(x_i - \frac{p_e\varphi}{2\pi}\right) \quad (11)$$

If [$\varphi_0$, $\varphi_4$] is the interval in which the φ angle varies, then in between the limits of this interval can be chosen three values $\varphi_1$, $\varphi_2$ and $\varphi_3$ which divide the initial interval in four other equal intervals. The arch of the arhimedic spiral from equation 1.11 correlated with the $\varphi \in [\varphi_0, \varphi_5]$ interval can be close enough approximated by the arch of a circle determined by the $\rho_j$ ($\varphi_j$), $j \in [1, 2, 3]$ co-ordinate points. The co-ordinates of this circle can be obtained by solving the following linear system:

$$2tg\alpha_e\left(x_i-\frac{p_e\varphi_j}{2\pi}\right)\sin\varphi_j\, a+2tg\alpha_e\left(x_i-\frac{p_e\varphi_j}{2\pi}\right)\cdot$$

$$\cdot\cos\varphi_j\, b+c=tg^2\alpha_e\left(x_i-\frac{p_e\varphi_j}{2\pi}\right)^2 \quad (12)$$

In which, the unknowns a, b, and c are connected with the co-ordinates of the circle's centre by using formulas in polar co-ordinates in the circle's equation.

## 1.2. THE RELIEF GRINDING OF CUTTING TOOLS THROUGH CIRCULAR PROCESSING

The gear cutters to which the present paper refers are detachable. Each gear' s cutting blade is attached on its body and fixed in a precise position connected with the gear' s axis. All the cutting tools, which are different from many points of view, have in common the following structure: a disk made of steel (the tool's body) built with canals in which the blades' tails are positioned; they are fixed with an exterior ring, made also from steel, by tightening-screws. Each gear' s cutting blade, being detachable, can be relief ground in two ways:

- Maintaining its position to the gear's axis. In this case the blade is processed while being attached to the tool. It' s necessary the cinematic processing of the relief ground surfaces, this means using cinematic chains for relief grinding which are specially designed for this process. This equals applying the principle of relief grinding face-mill cutters mono-block profiled.

- Changing the blade' s position to the gear' s axis by attaching it in a device that allows obtaining relief ground surfaces through their processing in universal machines. Obviously, if this position change is possible for a cutter' s blade, then it is possible for all the identical cutters, increasing in this way the number of the blades processed in a single change. We mean the circular processing of side surfaces and the plane processing of the superior surfaces. In the same time, although cinematically (geometrically) speaking, we approximate, this method would eliminate the negative dynamics of the method mentioned above.

## 1.3. THE CALCULUS OF THE DEVIATION FOR THE CIRCULAR RELIEF GROUND SURFACE

The replacement of the side surface with a conical surface is possible only if the difference between the two surfaces is less then an admissible limit. When the numeric calculus is made this limit is chosen to be the smallest tolerance field that is written on the original drawings of the gear cutters, that is 0.04 mm.

In a reference system where the Ox axis is the gear's cutter axis, in cylindrical co-ordinates, the equation of the helical relief ground side surface of an exterior blade is:

$$\rho_{elicoid}(x,\varphi)=tg\,\alpha_e\left(x-\frac{p_e\varphi}{2\pi}\right) \quad (13)$$

If the approximate cone' s axis has the co-ordinates:

$$\left.\begin{array}{l} y=y_1 \\ z=z_1 \end{array}\right\} \quad (14)$$

Where $y_1$ and $z_1$ were determined with the help of one of the methods shown previously, then the equation in cylindrical co-ordinates of the conical surface, having the peak semi-angle equal with $\alpha_e$ will be:

$$\rho_{con}(x,\varphi)=y_1\cos\varphi+z_1\sin\varphi-$$

$$-\sqrt{(y_1\cos\varphi+z_1\sin\varphi)^2-y_1^2-z_1^2+x^2tg^2\alpha_e} \quad (15)$$

With the help of 14 and 15 formulas the differences that appear between a circular processed surface and a helical relief ground one can be calculated. If an appropriate divisions system is chosen for x and φ variables, within an interval equal with the height / the centre angle, related with the re-sharpening of the cutter' s blade, then this differences on radial direction can be expressed with the following formula:

$$\Delta\rho(x,\varphi)=\rho_{elicoid}(x,\varphi)-\rho_{con}(x,\varphi) \qquad (16)$$

From geometrical point of view the most suitable solution for relief grinding the side surfaces and the superior surfaces of the gear' s cutter blades for conical wheels with teeth positioned in an arch of a circle is the helical relief grinding; a known type of this kind of original cutters is called "Helixform", which suggests that the company manufactures them using this method. From cinematical and dynamic point of view, this method implies difficulties, that can not always be correctly solved in Romania.

The circular relief grinding compared with the helical one, introduces a certain geometrical deviation (very small), but removes all the technological difficulties mentioned for the latter. Its advantages plead for the using and the spread of the circular relief grinding of all the gear' s cutters for conical wheels with curved teeth.

## 1.4. THE CIRCULAR RELIEF GRINDING OF THE "HELIXFORM 9" CUTTERS

The formulas determined in the previous chapter will be adapted for the circular relief grinding of "Helixform" cutters, which equip the gear cutter of 9 inches. The theoretical values determined for this type of tool and this dimension would be then practically checked, with the help of an experimental device, built for the gear cutters of 9 inches.

- The values published in the company' s catalogue [4].

The nominal dimension d of the gear is:

d = 228,6 mm.

The opening between peaks P.W. is symmetrical relative to the nominal diameter and is equal with:

P.W. = 3,048 mm.

The profile angles $\alpha$ have equal values for the exterior cutters / interior cutter equal with:

$\alpha_e = \alpha_I = 22^0 30'$.

- The determined values

The $\alpha_v$ inclined angle of the superior blade is represented in the execution drawings of the cutters used in S.C Tractorul Braşov by the value:

$\alpha_v = 6^0$.

If the same cinematic law was used to relief grind both the superior blade and the side one for the maintenance of the diameter, it was determined that at the peak of the blade there is not a plane curve, but a spatial one; so the quota $\alpha_v = 6^0$ indicates an approximate plane relief grinding of the top blade.

The $p_e$ pitch of the director helices,

$P_e = 70$ mm.

In conclusion, the proximity of the two surfaces is expressed by the difference's of the surface radiuses values (in cylindrical co-ordinates), that were determined in the same interval. The consequence of the two surfaces' proximity is that for the difference between the radiuses we obtain by calculus values not bigger than 0.02 mm. If we replace the helical surface with a conical one, the changes in the blade's geometry that this change introduces are admissible because the determined variation of (max. 0.02 mm) is less than the tolerance field imposed on the execution drawings of these blades (0.04 mm and even grater).

## REFERENCES


1. Hollanda, D., *Determinarea pe cale analitică a profilului frezelor disc cu unghi de degajare pozitiv*, Buletinul tehnico-ştiinţific, C.J.S. Braşov Tom. II, Vol. 4, 1977;
2. Hollanda, D., *Aşchiere şi scule aşchietoare*, Editura Didactică şi Pedagogică, Bucureşti, 1982;
3. Târziu, H., *Aspecte geometrice ale detalonării elicoidale a cuţitelor capetelor de danturat roţi conice cu dinţi în arc de cerc*, Revista RECENT, Universitatea "Transilvania" din Braşov, 2001;
4. Târziu, H., *Cercetări teoretice şi experi-mentale privind detalonarea capetelor de danturat roţi conice cu dinţi curbi pe maşini de rectificat universale*, Teza de Doctorat, Universitatea "Transilvania" din Braşov, 2002;

5. Moraru, V., ş.a., *Maşini-unelte speciale*, Editura Didactică şi Pedagogică, Bucureşti, 1982;
6. * * * *Gleason bevel & hypoid gear design*, Gleason Works, 1972;
7. * * * *Hypoid Cutter Sharpener Gleason*;
8. * * * *Hypoid Generator Gleason nr. 106 şi nr. 122*.