NUMERICAL PROCESSING OF RADIOLOCATION SIGNALS

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***Abstract**: This study analysis the methods of utilizing a Discrete Fourier Transform in numerical processing of radiolocation signals through introducing fast calculation specific algorithms.*
Signal processing in modern radars is realized by using both general algorithms and specific algorithms depending on high – base survey signals, i.e. linear modulation frequency impulse, utilizing signal numerical processors. The survey signals and those reflected from targets are all analogical signals. In order to be numerically processed, the Fourier transform is employed, so that the reflected analogical signal is replaced by a finite number of samples, processed at discrete moments.

***Keywords**: radar, alogorithm, processor, signals.*

1. INTRODUCTION

Nowadays the high – resolution modern radiolocation requires numerical processing of signals [1].

This is based on using the **D**iscrete **F**ourier **T**ransform (DFT), the discrete correlation and discrete convolution that helped in introducing fast calculation algorithms.

For this, there have been realized specific algorithms to radiolocation in order to eliminate human operations that produce considerable delays. This has been possible through using high – base survey impulses with linear modulation frequency and hi – fi signal processors.

A method of increasing the resolution utilized in modern radars is the "fast numerical convolution".

In practical applications, analysis and numerical processing of signals are realized through introducing fast calculation algorithms for Discrete Fourier Transform, named **F**ast **F**ourier **T**ransform (FFT) algorithms, as well as for correlation and discrete convolution, and they have constituted a great step forward.

Utilizing those fast calculation algorithms on the background of a fast increase of numerical signal processors' performances, has allowed that analysis and numerical processing to be made in real time for signals of higher and higher frequencies [6].

In the case of radar systems signals, starting from general usage algorithms, specific algorithms have been elaborated for real – time processing of this class of signals.

The most important algorithms [7] in this category are as follows:
- algorithms for adapted numerical filtering of high-base reflected signals;
- algorithms for automatic discovery of targets on a background of unintentional perturbations or intentional jamming;
- algorithms for dynamic calculation of threshold limit, used in the automatic discovery process for maintaining constant a false alarm value;
- algorithms for automatic determination of spatial coordinates and, based on those, of the main movement parameters for the discovered targets;
- algorithms for automatic selection of mobile targets on the background of reverberations from fixed targets and passive jamming;
- algorithms for radar protection against various sorts of jamming;
- algorithms for automatic following of mobile targets;
- algorithms for automatic classification of the discovered targets.

In modern radars, all the general algorithms and some of the specific signal processing algorithms process high – base survey signals like the **L**inear **M**odulation

Frequency (LMF) impulse [4].

The most prevalent practical application of those algorithms is the use of signal numerical processors, in fact micro – processors with high calculation capacity and a limited number of instructions.

Any signal processing requires a small number of simple arithmetical operations that repeat and that follow the next sequences (steps):

- reading a number from the operative memory;
- multiplying this with another number;
- adding the multiplication result with another number available from a working registry;
- writing the final result in the operative memory [5].

2. USING THE FAST FOURIER TRANSFORM IN NUMERICAL PROCESSING OF RADIOLOCATION SIGNALS

In all radiolocation systems the survey signals and those reflected from targets are all considered analogical signals [2].

In order to be numerically processed, the Fourier transform is employed, so that the reflected analogical signal, of margined support x(t*)*, is replaced with a finite number of samples N, and processed at discrete moments nT. Therefore, a discrete sequence x(nT) is formed, where T represents the time – sampler discreteness interval, called the "sampling period".

In keeping with the theory of sampling, for this replacement to be performed without loss of information, two conditions must be simultaneously fulfilled [3].

Firstly, the analogical signal spectrum must be of margined support:

$$X(f) = 0\ ;\ |f| > f_M \tag{1}$$

where f_M represents the maximum frequency within the analogical signal spectrum. Otherwise, the frequency limitation of the spectral function X(f) is done through "cross down" filtering of the x(t) signal.

Secondly, the period of the sampler must fulfill the Nyquist condition:

$$T \le \frac{1}{2f_N} \tag{2}$$

If considering a discrete periodical signal x(nT), of NT period, its DFT is defined as being the sequence of values x(kF), periodical, of an NF period that is given by the relation:

$$x(kF) = \mathcal{F}_D[x(nT)] = \sum_{n=0}^{N=1} x(nT)\, e^{-j\frac{2\pi}{N}k_n}; \quad k = 0,1......N-1 \tag{3}$$

The measure F represents the frequency discreteness interval of the spectral function, determined by relation (5), and it depends on the maximum frequency of this spectral function.

Relation (3) makes it possible to determine the discrete spectrum suitable for a limited duration discrete signal.

Discrete Fourier Transform Inverse of the x(kF) sequence is by definition the sequence of values given by the relation:

$$x(nT) = \mathcal{F}^{-1}[x(kF)] = \frac{1}{N}\sum_{k=0}^{n-1} x(kF)\, e^{j\frac{2\pi}{N}k_n}; \quad n = 0,1......N-1\ . \tag{4}$$

The above relation allows deducing the wave shape whenever its discrete spectral function is known.

The discreteness in N points of the spectral function involves a frequency sampling interval equal to:

$$F = \frac{2f_M}{N} \tag{5}$$

In case the Nyquist relation (2) is fulfilled just on the line:

$$T = \frac{1}{2f_M} \tag{6}$$

then, from expression (5) the final value for the frequency discreteness interval can be deducted:

$$F = \frac{1}{N\,T} \tag{7}$$

In the above relation, T_0 = NT represents the discrete sequence duration x(nT), formed of N samples.

The spectral functions of limited duration signals are not of margined support. Because of that, they have to be limited by using spectral windows with margined support.

Relation (6) shows that the resolution of discreteness operation in time for analogical signal is the higher as the maximum frequency f_M, with its spectral function X(f) undertaking limitation, has a higher value.

By employing the DFT algorithm, it can be assured discrete approximation of a Fourier transform for analogical signal, by covering the following stages:

- it is chosen an analogical signal x(t) with a finite duration T_0, having a spectral function X(f) of margined support;
- through its sampling with the period $T = \frac{T_0}{N}$ the sequence x(nT) that contains N values is obtained;
- the following relation is used:

$$x_a(kF) = T\,\mathcal{F}_D[x(nT)] \tag{8}$$

for obtaining a discrete spectral function X(kF);

- using Discrete Fourier Transform (DFT) inverse, there can be obtained N samples of the discrete signal x(nT) from the N samples of the approximate spectral function, according to the relation:

$$x(nT) = \frac{1}{T}\mathcal{F}_D^{-1}[x_a(kF)] \tag{9}$$

However, the spectral analysis of analog signals by using the DFT triggers a series of errors.

By adequately choosing resolution T in the time domain dependent on f_N, or resolution in the frequency domain F dependent on T_0, a precisely enough calculation can be guaranteed.

If in the definition relations (3) and (4) of DFT direct and, respectively, inverse, the following substitution is used:

$$W_N = e^{j\frac{2\pi}{N}} \tag{10}$$

and for simplification, the sequences x(nT) and X(kF) are replaced with sequences x(n) and X(k), the final expressions of direct and inverse DFT become:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{-kn}; \quad k = 0,1,....N-1 \tag{11}$$

and

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kn}; \quad n = 0,1,....N-1 \tag{12}$$

If in the relation (11) the amount shown in the right member is explicitly written, it can be observed that for obtaining a single term of the discrete spectral function:

$$X(k) = x(0)W_N^0 + x(1)W_N^{-k} + ..x(N-1)W_N^{-(N-1)k} \tag{13}$$

there are performed a total of N multiplications and N − 1 additions in complex.

For values of ×1000 – class for N, useful in various numerical signal processing practical applications, unacceptably large values for N^2 do result.

Determining the N terms of DFT direct involves N^2 multiplications and, respectively, N(N–1) additions in complex, of $\times 1000^2$– class.

In order to reduce the amount of calculations to an allowable level, special algorithms are elaborated and utilized, known as the **Fast Fourier Transform**.

For fast – calculating the Discrete Fourier Transform, the classic algorithm named **Fast Fourier Transform with time decimation** is employed.

The time – decimation of a temporal numerical sequence requires a reordering process of the sequence terms according to a certain ordering criteria, defined to begin with.

The temporal sequence x(n) with n = 0, 1,....N–1 and $N = 2^M$, which separates the samples with an even index from those with an odd index:

$$x_1(n) = x(2n)\ ; \quad n = 0,1,....N/2-1$$

$$x_2(n) = x(2n+1) \tag{14}$$

Through separating even and odd sequences, the following relations occur:

$$X(k) = X_1(k) + W_N^{-k} X_2(k);$$

$$X\left(k + \frac{N}{2}\right) = X_1(k) - W_N^{-k} X_2(k); \tag{15}$$

$$k = 0,1,....N/2-1$$

The above expressions represent the essence of the RFT algorithm with time-decimation.

The **first stage** of decimation for an initial sequence allows the determining of a DFT in N points through combining two DFTs in N/2 points.

If the spectral functions $x_1(k)$ and $x_2(k)$ are known, for determining the spectral function x(k) there is required a total of N multiplications and, respectively, N additions in complex.

It is, therefore, favorable that the product $W_N^{-k}X_0(k)$ to be calculated only once for each k, and the result to be used in both relations, thus ensuring the reduction of multiplications number down to the N/2 value.

In the **second stage** of division, spectral functions $X_1(k)$ and $X_2(k)$ in N/2 points, are each calculated by combining two DFTs each in N/4 points.

If the four DFTs are supposedly known, and this stage necessitates also N/2 multiplications and N additions in complex, then the successive division by two of the sequences is repeated m times.

The **last stage** of division determines a number of N/2 DFTs, in two points, and necessitates the same number of multiplications and additions as the previous stages.

As a result, for calculating a DFT in $N = 2^m$ points, through the FFT algorithm with time – decimation, the following operations in complex are required:

$$N_x = \frac{N}{2}m = \frac{N}{2}\log_2 N\,; \quad N = 2^m$$
$$N_x = Nm = N\log_2 N \tag{16}$$

For larger values of N, the above relation provides a considerable reduction in the amount of initial calculations.

Figure 1 represents the graph for an FFT algorithm with time decimation for calculating a DFT in N = 8 points.

Each arrow corresponds with a multiplication operation of signal from the angle side's origin with the specified term besides the arrow, and each node corresponds to an addition operation of convergent signals in the same node.

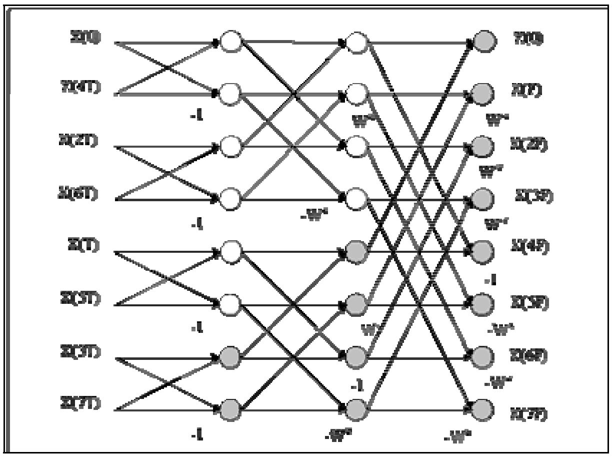

Fig. 1 FFT algorithm with time decimation for calculating DFT in N = 8 points graph

This graph allows to be determined the modifications undertaken by partial and final results of the FFT algorithm when the input sequence moves with only one pattern or one pitch. This situation occurs when it is performed the DFT calculation of successive sequences within signals that partially overlap, which is often met in practice.

In figure 2 is represented the simplified graph of FFT in eight points, when the input sequence displaces with one pitch.

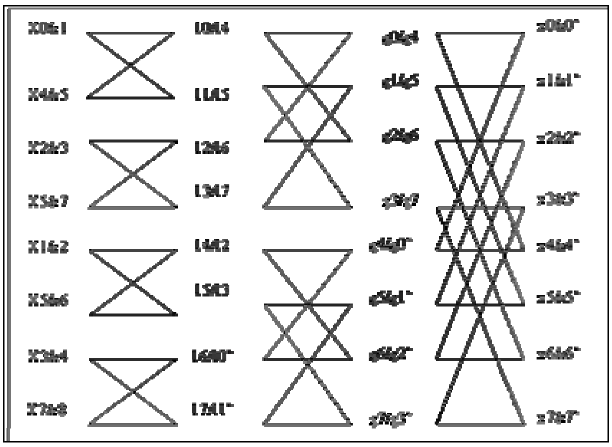

Fig. 2 FFT with displacement in 8 points graph

Consequently, the total amount of calculation for FFT with displacement, noted FFTD, is:

$$N_x = 2^m - 1 = N - 1;\ \ N = 2^m$$
$$N_+ = 2(2^m - 1) = 2(N - 1) \tag{17}$$

By using the relations (16) and (17), it can be deduced how many times is the necessary amount for FFT bigger, as against the amount of calculations of FFT with displacement (FFTD):

$$\frac{N_x^{TFR}}{N_x^{TFRD}} = \frac{N_+^{TFR}}{N_+^{TFRD}} = \frac{N_m}{2(N-1)} = \frac{m}{2}; \quad m>4 \qquad (18)$$

For larger values of m, reducing the amount of calculations arisen from relation (18) is considerable: for example, when N = 1024 for which m = 10, FFTD requires a five times less number of operations in complex toward FFT.

In the case that the input sequence displaces two or more steps, relation (18) does not remain valid any longer.

Reducing the amount of calculations obtained by using the mono-dimensional FFTD becomes insignificant.

Eliminating this disadvantage of FFT with displacement can be realized by using bi-dimensional FFT algorithms. Those algorithms are based on transforming the mono-dimensional input sequence x(n), containing $N = 2^m$ terms, into a bi-dimensional table with N_1 rows and N_2 columns.

The number of rows and columns of the bi-dimensional table must be exponents of 2:

$$N = 2^m = 2^{p+r}; \; N_1 = 2^p \quad N_2 = 2^r \qquad (19)$$

A similar table can be obtained by fragmenting the initial sequence x(n) in N_2 subsequences considered in natural order, each having N_1 terms.

The N_2 subsequences thus obtained form the columns of the bi-dimensional table.

The table can be realized by using the following variable changes:

$$\begin{aligned} N &= n' + N_1 n''; \quad n' = 0, 1, \ldots N_1 - 1 \\ & \qquad\qquad\quad\;\; n'' = 0, 1, \ldots N_2 - 1 \\ k &= N_2 k' + k''; \quad k' = 0, 1, \ldots N_1 - 1 \\ & \qquad\qquad\quad\;\; k'' = 0, 1, \ldots N_2 - 1 \end{aligned} \qquad (20)$$

DFT in $N = N_1 \times N_2$ points can be determined by following the next steps:

- DFT is calculated in N_2 points for each of the N_1 lines of the bi-dimensional table;
- each of the spectral components is multiplied and calculated with the corresponding exponentials of $W_N^{-n'k''}$ shape;
- it is calculated N_2 DFT in N_1 points.

Figure 3 represents the bi-dimensional method of handling a mono-dimensional DFT, for the case of a sequence with N = 32 terms, considered a matrix with N_1 = 4 lines and N_2 = 8 columns.

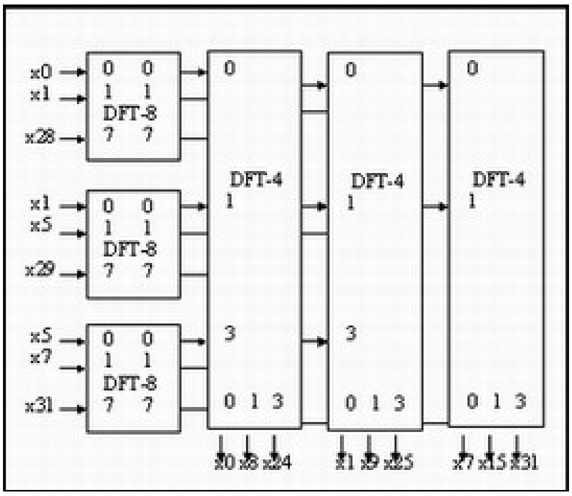

Fig. 3 Bi-dimensional calculation of a mono-dimensional DFT

The N_1 DFT in N_2 points and the N_2 DFT in N_1 points can be calculated using the FFT algorithm with time – decimation.

In this case, using the relation (16) and taking into account that the number of lines and columns is given by relation (18), the volume of calculation of the bi-dimensional FFT algorithm can be obtained:

$$\begin{aligned} N_x &= N_1\left(\frac{N_2}{2} r\right) + N_2\left(\frac{N_1}{2} p\right) + N = \frac{N}{2} m + N \\ N_+ &= N_1(N_2 r) + N_2(N_1 p) = Nm \end{aligned} \qquad (21)$$

If comparing the relations (21) and (16), results that for calculating a DFT in N points, for the same number of additions, the bi-dimensional DFT algorithm requires N multiplications moreover against the FFT algorithm with time – decimation.

From the sequence x(n) with N elements, disposed as a bi-dimensional table of $N_1 \times N_2$ type, a new sequence can be created x'(n) same with N elements, arranged as a table with the same dimensions.

In this second table, the first $N_2 - 1$ columns coincide with the last $N_2 - 1$ columns of the initial table and the N_2 column is new.

For the particular case represented in figure 3, replacing the sequence x(n) with sequence x'(n) is equivalent to displacing a step up the terms of the subsequences from the entrance of each of the $N_1 = 4$ DFT in $N_2 = 8$ points. For example, the subsequence x_0, x_4, x_0, ... x_{20}, transforms into subsequence x_0, x_8, x_{16}, ... x_{32}, whenever the initial sequence x_0, x_1, ... x_{31} is replaced with the new sequence x_4, x_5, ... x_{35}.

By using the relation (17) it can be deduced the calculation volume required for the bi-dimensional FFT algorithm with displacement:

$$N_x = N_1(N_2 - 1) + N_2\left(\frac{N_1}{2}p\right) + N =$$

$$= N\left(\frac{p}{2} + 2\right) - N_1$$

$$N_+ = 2N_1(N_2 - 1) + N_2(N_1 p) =$$

$$= N(p + 2) - 2N_1 \qquad (22)$$

From the above relations it can be deducted the fact that, reducing the amount of calculations by using bi-dimensional FFT with displacement is the greater the number of samples $N_1 = 2^p$ with which the input sequence moves, has more reduced values [3].[]

3. CONCLUSION

In this paper, the Discrete Fourier Transform has been introduced, and two FFT base algorithms have been analyzed:
- the FFT algorithm with time – decimation;
- the FFT bi-dimensional algorithm.

These constitute the first steps in numerical processing of radiolocation signals. The purpose of employing those algorithms is the substantial reduction, at an admissible level, of the calculation volume (multiplications and additions) of DFT and implementation in time – domain of adapted numerical filters or signals numerical processors [8].

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