MODELING IN OPERATIONAL RESEARCH

Serjiu-Gheorghe OBREJA, Gavrilă CALEFARIU

"Transilvania" University of Braşov, Romania (obreja.serjiu@afahc.ro, gcalefariu@unitbv.ro)

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Abstract: *In order to be able to develop the organization at a high level of efficiency, every structure that makes up the organization should grow relatively at the same pace as the other structures or compartments.*

Hence the need to model the organizational system at the virtual level and to simulate the different stages of the organization system development taking into account the disturbing factors caused by the internal and the external environment of the organization.

Keywords: *operational research, mathematical modeling, model validation, implementation*

1. INTRODUCTION

As we know, operational research is based on the mathematical analysis of the various internal and external environmental issues of the organizational society as a whole, modeling and validating various organizational issues by finding an optimal solution.

The way they are defined, operational research has constituted the basis for the development of the organizational culture as a whole, since there have been identified many problems that have arisen from the multi-level segregation of work in an organizational unit.

In order to be able to develop the organization at a high level of efficiency, every structure that makes up the organization should grow relatively at the same pace as the other structures or compartments. However, experience says that what is good for a compartment may be to the detriment of other compartments or structures of the organization. Thus, the allocation of resources, which is more and more difficult to achieve in case of a complex organization, is streamlined, it leading ultimately to an increase in the efficiency of the organization by intersecting the goals of all compartments or structures.

Hence the need to model the organizational system at the virtual level and to simulate the different stages of the organization system development taking into account the disturbing factors caused by the internal and the external environment of the organization.

2. HISTORICAL REFERENCE

Operational research has achieved an impressive progress during the Second World War at the same pace with the increasing complexity of military operations, from the tactical level to the operational and strategic level.

Thus, in support of the Armed Forces, top researchers from all areas were invited to model and simulate various complex stages to make military action more effective, taking into account several external factors of political, economic and social nature.

All these challenges, which the operational research attempts to solve are due to the influence of environment on the organization and are aimed at finding solutions to the increasingly complex challenges of the organizational environment challenges that originate in the unprecedented development of industry and applied sciences.

After World War II, operational research has developed primarily due to the migration of the majority of specialists in the field of military operational research in the field and respectively in the civilian economic and social environment. The industrial boom at the end of the Second World War, based on various economic and social recovery plans for the defeated nations, has generated similar operational research problems in the civilian organizational environment, and military specialists in operational research have been the ones to settle the foundations of operational research in the civilian field.

The organizational culture benefited from the help of military specialists, but a significant contribution to it was facilitated by the unprecedented technological revolution in the IT area. The increased computing power of electronic computers has shortened the arithmetical computational time in the simulation process, which has led to increased work productivity in the field of operational research.

Nevertheless, the real change occurred in the early 1990s and culminated in the early 21st century when personal computer development accelerated, and the development of software packages dedicated to Microsoft Excel operational research began to provide solutions to almost all new challenges in the operational research environment.

The last cornerstone was the development of portability by using portable devices such as laptops, notebooks or personal assistants used on hardware architectures that use multicore processors with artificial intelligence.

3. WHY OPERATIONAL RESEARCH?

Definition: *Operational research is the study of mathematical models of complex engineering and management problems and perspectives of implementation and use of possible optimal solutions in this study* [1].

Because, as the title says, when there are concerns or research on improving the organizational culture, they need to be operationalized, or else the problem formulated by careful observation must be solved by constructing a mathematical model that will reveal the essence of the problem or the hypothesis from which it started.

Therefore, not only the problem has to be solved, but also the sum of unknown factors that derive from the phases of the very modeling in the operational researches; subsequently we can enumerate 4 phases of modeling (FIG. 1):

1. Knowledge of reality in the studied organization, in order to improve the decisional information mechanism, to define the issue of interest and to collect important data.
2. Formulation of a mathematical model of problem representation. This operation, in most of practical cases, consists of applying a classical modeling tool chosen from the extremely varied range available to us due to the theory of operational research. In such situations, the analyst’s ability is to establish the correspondence between the reality and the modeling tool known in the specialized literature.
3. Development of a procedure or technique based on computer simulation conclusions in order to find solutions to the mathematically modeled problem.
4. Testing and implementing the model, refining the results and making decisions.

An axiom-based model (axiomatic system) comprises:

- the axioms of the system, representing sentences expressed in mathematical form, usually very few, which contain some very general truths on the phenomenon that is being modeled, so general that all concrete and particular findings can be deduced from the general ones;

- rules of inference, representing rigorous prescriptions, the only ones admitted in the system, from which one passes from axioms to theorems, or from already demonstrated theorems to new ones;
- theorems, i.e., more or less particular sentences, mathematically expressed, deduced by step-by-step inference rules from axioms and expressing the properties of the modeled phenomenon.

When in the axiomatic modeling process the concepts to be used are defined in a limiting manner, i.e. a list of the mathematical notions and operations admitted in the system is given from the beginning; a superior form of axiomatic system called formal system is obtained.

Formal systems are still very little used in science and even less in the disciplines of organization and economic leadership.

Axiomatization and, ultimately, formalization, represents the future in mathematical modeling, due to the exceptional rigidity they introduce, the considerable diminution of the elements of intuition and arbitrariness, which, although much less than in the non-mathematical models, are still present in the axiomatized mathematical modeling .

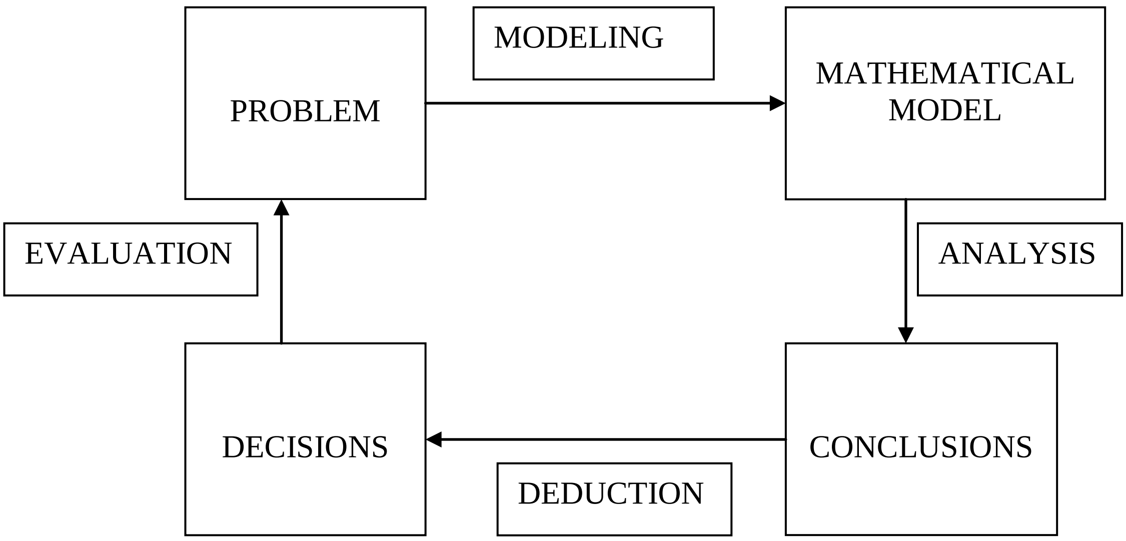

FIG. 1 Operational research method

4. CHARACTERISTICS OF MODERN MODELING INSTRUMENTS

Given the fact that we know the decision problem, the process will begin with formulation or modeling. We define the variables and quantify the relationships necessary to describe the relevant behavior of the organizational system. Then the analysis comes. We apply our mathematical knowledge and skills to see what the mathematical model suggests. Note that these conclusions are extracted from the model, not from the problem it is intended to represent. In order to complete the process, we must participate in inference, that is to say that the conclusions drawn from the model are sufficiently significant to deduce the decisions for the person or persons who wish to solve this problem.

Often an evaluation of the decisions deducted in such a manner proves to be inadequate or extreme and cannot be used for implementation. Permanent thinking leads to revised modeling, in a continuous loop.

Thus, the definition of the problem becomes the most important phase, defining the new operational requirements of the organization on the basis of the hypothesis.

This process of defining the problem is crucial, because it is very important how it affects us and how relevant the conclusions of the study are.

After defining the problem of the decision-maker, the next step is to reformulate this problem in a form that is convenient for analysis. The conventional approach to operational research is to build a mathematical model that represents the essence of the problem. Before discussing how to formulate such a model, we first analyze the nature of models, in general, and the mathematical models, in particular.

Models or idealized representations are an integral part of everyday life. Common examples include aircraft models, portraits, globes, and so on. Similarly, models play an important role in science and business, illustrated by atom models, genetic structure models, mathematical equations describing physical motion laws or chemical reactions, graphs, organizational diagrams, and industrial accounting systems.

Such models are invaluable for abstracting the subject matter of the investigation, presenting interrelationships and facilitating the analysis.

The next step is to check if the mathematical model is accurate enough, reflecting the initial hypothesis in order to define the real problem.

Next, validation of the mathematical model must be achieved by experiments leading to the validation of the hypothesis or modification of the model so that the results can verify these assumptions. This part, which is also called validation of the model, is in fact the pure scientific part of the operational research, in general.

After validating the mathematical model, clear conclusions should be drawn to help decision-makers in the organization when the case is. Of these solutions, we have to find the optimal one from the multitude of solutions that exist and this optimal solution must best solve the problem of the hypothesis or the problem of the organization.

Thus, searching for the optimal solution or optimal solution theory is a predilection theme in operational research.

Next, we will try to demonstrate to what a clear definition of the problem we want to solve is being reduced how it is achieved, in contrast to the data acquisition meant to define the hypothesis [2].

Thus, if there are *n* measurable decisions about them, they are represented as decision variables (for example $x_1; x_2 ... x_n$) whose values must be determined. The appropriate measure of performance (e.g. impact) is then expressed as a mathematical function of these decision variables (e.g., I = $3x_1 + 2x_2 + ... + 5x_n$). This function is called the objective function. Any restrictions on the values that can be attributed to these decision variables are also expressed mathematically, usually through inequalities or equations (for example, $x_1 + 3x_1x_2 + 2x \le 10$). Such mathematical expressions for restrictions are often called constraints. The constants in the formulas that define the **constraints** and the objective function are called the model **parameters**.

The mathematical model should ultimately indicate that the problem is to choose the values of the decision variables to maximize the objective function, subject to the specified constraints.

Such a model and its minor variations are suitable in the study of operational research of the type studied in the management of the organization.

Determining the appropriate values to be attributed to model parameters (one value per parameter) represents both the critical part and the challenge part of the model building process, which will ultimately aim at parameterizing the mathematical model.

Unlike problems in school books, where the mathematical model is parameterized, determining the parameter values for real issues requires collecting relevant data.

Data collection is a fairly difficult part because import and data collection are time-consuming. Therefore, the value assigned to a parameter is often only a gross estimate. Because of the uncertainty about the true value of the parameter, it is important to analyze how the solution deriving from the model would change (or would not change) if the value assigned to the parameter would be modified to other real values.

This process is called sensitivity analysis. Although we refer to the mathematical "model" of a business, for example, a real business normally does not have a "correct" mathematical model. The testing process of a model usually leads to a succession of "fair" models that provide a better and better representation of the problem. It is possible for two or more completely different models to be developed to help analyze the same problem. A particularly important type of mathematical model is the linear programming model, in which the mathematical functions occurring in both the objective function and the constraints are all linear functions [3].

The specific linear programming models are built to suit a variety of issues such as:

- determining the combination of products that maximize profit in hypermarkets;
- effective projection of radiotherapy that fills a tumor, while reducing damage to healthy tissues near the tumor, in medical cases in oncology;
- crop area allocation that maximizes total net profitability and improves agricultural production;
- a combination of pollution reduction methods that achieve air quality standards at a minimal cost in large metropolitan and urban agglomerations.

5. EXAMPLES REGARDING MATHEMATICAL MODEL

5.1 General modeling example

In order to have an overview of a simple mathematical description of a general state system problem, which adequately predicts the response of the physical system to all anticipated inputs, we can describe the system using a normal differential equation as follows:

$$x_1(t), x_2(t), \ldots, x_n(t) \tag{1}$$

With the state variable at time *t*:

$$u_1(t), u_2(t), \ldots, u_m(t) \tag{2}$$

The restrictions of this process, at time *t*, may be described through a differential equation of nth order, as follow:

$$\begin{aligned}
\dot{x}_1(t) &= a_1(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), t) \\
\dot{x}_2(t) &= a_2(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), t) \\
&\;\cdot \\
&\;\cdot \\
&\;\cdot \\
\dot{x}_n(t) &= a_n(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), (t)).
\end{aligned} \tag{3}$$

Thus we will define as state vector:

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{4}$$

and control vector:

$$u(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \tag{5}$$

and so the state equation can be written:

$$x(t)=a(x(t),u(t),t) \tag{6}$$

5.2 Particular modeling example

In other words, if we wanted to model a real economic system using linear programming models to analyze the maximum efficiency of a complex economic system, we can use the structure of the general linear programming model, which is constituted primarily from the set of activities $\{A_1, A_2,...A_n\}$that make up the economic system, the resources used$\{R_1, R_2,...R_m\}$, as well as the multitude of technical and manufacturing relationships.

In order to link activities and resources, we must consider that this is determined by the production technology used by each activity $A_j\left(j=1,...,n\right)$ being characterized by the column vector $a^{(j)}$ with the components$\left(a_{1j}, a_{2j},..., a_{mj}\right)$.

The elements $\{a_{ij}, i=1,...,m; j=1,...,n\}$ are called technological coefficients or specific technological consumption coefficients and show how much of the resource R_i is consumed to produce a specific product unit P_j, within the activity A_j.

Column vectors or, better known, all manufacturing technologies can be organized in a matrix A with *m* lines and *n* columns where each line refers to an allocated resource $R_i(i=1,...,m)$and each of the columns refers to an activity $A_j\left(j=1,...,n\right)$.

Thus, we note with $x_j\left(j=1,...,n\right)$ the result of the activity A_j in a given period and with$b_i\left(i=1,...,m\right)$, the quantities available from the resources $R_i\left(i=1,...,m\right)$ and we write mathematically the following technological restrictions, as follows:

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m \end{array} \quad \text{or } A\cdot x \le b \tag{7}$$

$$\text{where } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{și } b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{8}$$

There are two assertions:

1. The amount of resource consumed may not exceed the quantity present at that time;
2. The total consumption R_{ij} of the resource R_i for the achievement of the activity A_j is proportional to the result of the activity, i.e. $x_j, \Rightarrow R_{ij} = a_{ij} \cdot x_j$.

The Restriction System (7) implements the linkage between resources and activities through linear restrictions m.

The linear programming model of a real particular economic system, as outlined above, must contain, in addition to the type (7) restrictions, a performance criterion that allows assessment of the effectiveness of each activity.

Depending on the purpose of modeling, we can choose as an efficiency criterion an indicator that measures the effort or an indicator that measures the outcome [4].

We can also choose as a criterion a ratio between the result and the effort or effort per result, depending on the purpose of the mathematical modeling of the real economic system.

Thus, we have the linear function:

$$f(x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \tag{9}$$

which evaluates the efficiency of any real economic system, we obtain the following linear programming program:

$$\underset{x}{optim}\left[f(x)\right] \tag{10}$$

$$\begin{cases} \left.\begin{aligned} \sum_{j=1}^{n} a_{ij} \cdot x_j \le b_1, i \in I_1 \\ \sum_{j=1}^{n} a_{kj} \cdot x_j \ge b_k, k \in I_2 \end{aligned}\right\} \quad I_1 \cup I_2 = \{1,2,\dots,m\} & (11) \\ x_j \ge 0,\ j = \overline{1,n} & (12) \end{cases}$$

Where:

1. Relation (10) is the objective function of the efficiency of the programming problem;
2. Relationship (11) represents resource-type restrictions and qualitative technical-economic restrictions;
3. Relationship (12) is the condition of non-negativity of the variables that actually provide an economically achievable solution.

CONCLUSIONS

Mathematical models have multiple advantages over a verbal description of the problem. One advantage is that a mathematical model describes an issue in a much more concise manner, taking into account internal and external environmental factors. This tends to make the general structure of the problem more understandable and helps to find important cause-effect relationships. In this way, it is more clearly indicated which additional data is relevant for the analysis. It also facilitates solving the problem by taking into account all its interrelationships at the same time. Finally, a mathematical model establishes a link to the use of high-power software-mathematics techniques and computers to analyze the problem.

Indeed, to solve many complex mathematical models, simulation-software, as MATLAB or MAPLE, designed for both personal computers and mainframe computers has become widely available.

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