A METHOD FOR SYNTHESIS OF FAMILIES OF SETS OF PHASE MANIPULATED SIGNALS WITH OPTIMAL CORRELATION PROPERTIES

Dilyan DIMITROV, Borislav BEDZHEV, Monika BEDZHEVA

"Vasil Levski" National Military University - Artillery, Air Defense and CIS Faculty, Shumen, Bulgaria

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Abstract: *In the paper a method for synthesis of families of sets of phase manipulated signals with optimal correlation properties is substantiated. The method could be used in the development of new radar sensor networks, which exploit effectively the electromagnetic spectrum and can operate practically without self-interferences and multiaccess interferences.*

Keywords: *radio communication system, signal processing, synthesis of phase manipulated signals.*

1. INTRODUCTION

Recent years have seen a steady increase in the number and type of wireless devices used in all areas of the society. For example, industrial networks, where reliability requirements are very high, are constantly being upgraded with control and monitoring applications as well as new sensor and smart networks are being implemented [1], [2]. The military [3] and civilian [4] consumer wireless, cellular and satellite networks are also in rapid progress.

The analysis of these examples show that the importance of different types of wireless electronic communication services increases with relatively equal opportunities to provide the limited natural resource - electromagnetic spectrum. Accounting this situation in the paper a method for synthesis of families of phase manipulated (PM) signals with optimal correlation properties is proposed. The method could be used in the development of new radar sensor networks, in which deleterious effect of the self-interferences (SIs) and multiaccess interferences (MAIs), caused by the multipath radio waves propagation and by the simultaneous use of the communication channel by many users respectively, is minimal [3], [5], [6].

The paper is organized as follows. First, the basics of the signal processing in the receivers of the radar systems are recalled. After that the properties, which the PM signals ought to possess in order to minimize SIs and MAIs, are systematized. On this base a method for synthesis of families of PM signals with optimal correlation properties is developed. At the end, the applications of the proposed method are analysed.

2. BASICS OF THE SIGNAL PROCESSING IN THE RECEIVERS OF THE RADAR SYSTEMS

In the general theory of communication systems it is proved that the communication receivers must be a filter, matched to the signal, which have to be detected. This conclusion ensues from the following circumstances.

First, most often in communication systems the signals must be detected in the presence of the so-called additive white Gaussian noise (AWGN).

Second, the receiver construction as a matched filter (MF) maximizes the signal-to-noise ratio (SNR) in the receiver output, when the signals are influenced by AWGN.

Third, the situation, when the background noise is AWGN, is considered as one of the worst cases of signal reception. In other words, if the receiver is a MF, but the background noise is not AWGN, then the performance of the receiver most probably is better.

Due to the above conclusion, most often, the digital signal processing in the receiver of a radar can be described by the following polynomial model [5], [6]:

$$[\sum_{i=0}^{N-1} \mu(i)x^i][\sum_{i=0}^{N-1} \mu^*(i)x^{-i}] = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^r \ . \tag{1}$$

In (1) the following notations are used.

First, the digital signal

$$\{\mu(i)\}_{i=0}^{N-1} = \{\mu(0), \mu(1), \dots, \mu(N-1)\} \tag{2}$$

consists of the samples of the digitalized echo-signal. As the echo-signal is produced by reflection of the probe signal from the surface of some object, the echo-signal is a diminished copy of the probe signal of the radar transmitter [3], [5], [6]. Due to this reason, the samples (2) are complex numbers, presenting the complex envelopes of the elementary phase symbols (or chips) with duration τ_{ch}, forming the probe signal.

Analogously, the sequences of complex numbers

$$\{\mu^*(i)\}_{i=0}^{N-1} = \{\mu^*(0), \mu^*(1), \dots, \mu^*(N-1)\}, \tag{3}$$

$$\{P_{\mu\mu}(r)\}_{r=-N+1}^{N-1} = \{P_{\mu\mu}(-N+1), \dots, P_{\mu\mu}(-1), P_{\mu\mu}(0), \dots P_{\mu\mu}(N-1)\}, \tag{4}$$

are the samples of the finite response matched filter (MF), used in the radar receiver, and the autocorrelation function (ACF) of the digital signal (2) respectively.

In (3) the symbol "*" stands for "complex conjugation".

Second,

$$F_\mu(x) = \sum_{i=0}^{N-1} \mu(i)x^i \tag{5}$$

is the so-called generating function or Hall polynomial, associated with the digital signal (2).

It should be pointed out that the powers of the variable x^{-i}, x^i denote "overtaking or delay at i time-clocks with duration τ_{ch} during the signal processing" respectively.

Analogously, the generating functions (Hall polynomials) of the digital signals (3) and (4) are

$$F_\mu^*(x^{-1}) = \sum_{i=0}^{N-1} \mu^*(i)x^{-i} \ , \tag{6}$$

$$F_{P_{\mu\mu}}(x) = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^r \ . \tag{7}$$

Now it should be taken in consideration, that performance quality of the radar sensor networks is worsened mainly by the SIs and MAIs [3], [4], [5], [6].

The SIs can be explained as follows. The most often in the radar receiver an input mix of echo signals, reflected by different objects, is present. In such situations, the ACF side-lobes of echo-signals of more huge objects can mask the ACF main lobes of echo-signals of smaller objects, which may be more important.

The MAIs are result of the simultaneous work of many radar sensors and other radio electronic devices in a common frequency band.

Accounting the deleterious effects of SIs and MAIs, the rest part of the paper is focused on the development of method for synthesis of families of sets of PM signals with optimal correlation properties.

The theoretical and practical importance of this problem ensues from the following advantages of these families of signals.

First, the PM signals can be generated by simple, reliable and cost-effective digital devices.

Second, the sets of PM signals are wideband as they exploit sets of $M, M \geq 2$ carrier frequencies. As a result, the spectral density of signals can be very small, which provide electromagnetic compatibility with other radio communication systems as well as possibility of reuse of the frequency resources.

Third, the aggregated ACF of every set as well as the aggregated cross-correlation function (CCF) of every pair of sets of PM signals resemble a delta-pulse, i.e. their side-lobes are small or are completely absent.

3. A METHOD FOR SYNTHESIS OF FAMILIES OF SETS OF PHASE MANIPULATED SIGNALS WITH OPTIMAL CORRELATION PROPERTIES

Let the matrices G and H

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ g_{M1} & g_{M2} & \cdots & g_{MM} \end{bmatrix}, H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ h_{M1} & h_{M2} & \cdots & h_{MM} \end{bmatrix} \quad (8)$$

be two (not necessarily different) unitary orthogonal matrices, which sizes are $M, M \geq 2$. This means that all entries of G and H have unit magnitude and their columns and their rows are orthogonal, i.e.:

$$\begin{bmatrix} g_{1n_1} \\ g_{2n_1} \\ \cdots \\ g_{Mn_1} \end{bmatrix} \otimes \begin{bmatrix} g^*_{1n_2} \\ g^*_{2n_2} \\ \cdots \\ g^*_{Mn_2} \end{bmatrix} = C_{gn_1} \otimes C^*_{gn_2} = \\ = \sum_{m=0}^{M-1} g_{mn_1} g^*_{mn_2} = \begin{cases} M, n_1 = n_2, \\ 0, n_1 \neq n_2, \end{cases} \quad (9)$$

$$\begin{bmatrix} h_{1n_1} \\ h_{2n_1} \\ \cdots \\ h_{Mn_1} \end{bmatrix} \otimes \begin{bmatrix} h^*_{1n_2} \\ h^*_{2n_2} \\ \cdots \\ h^*_{Mn_2} \end{bmatrix} = C_{hn_1} \otimes C^*_{hn_2} = \\ = \sum_{m=0}^{M-1} h_{mn_1} h^*_{mn_2} = \begin{cases} M, n_1 = n_2, \\ 0, n_1 \neq n_2. \end{cases} \quad (10)$$

In (9) and (10) the symbol "$\otimes$" stands for the ordinary scalar product of vectors. Besides, C_{gn_1}, $C^*_{gn_2}$, and C_{hn_1}, $C^*_{hn_2}$ are the n_1-th column and the complex conjugate of the n_2-th column of the orthogonal matrices G and H respectively.

Analogous property possess the rows of the orthogonal matrices G and H.

Here it should be noted that in general theory of matrices several methods have been developed, by the means of which unitary orthogonal matrices with arbitrary sizes can be synthesized [6].

From the orthogonal matrices G and H a family of M derivative matrices can be obtained when the columns of the matrix G are multiplied consecutively by the rows of the matrix H. The general form of the derivative matrices, forming the matrix family, is

$$D_m = \begin{bmatrix} h_{m1}g_{11} & h_{m2}g_{12} & \dots & h_{mM}\, g_{1M} \\ h_{m1}g_{21} & h_{m2}g_{22} & \dots & h_{mM}\, g_{2M} \\ \dots\dots & \dots\dots & \dots & \dots\dots \\ h_{m1}g_{M1} & h_{m2}g_{M2} & \dots & h_{mM}\, g_{MM} \end{bmatrix}, m = 1, 2, \dots, M \tag{11}$$

As seen, in (11) $\{h_{m1}, h_{m2}, \dots, h_{m2}\}$ is the m-th row of the matrix H.

Proposition: Let the family of matrices $\{D_m\}_{m=1}^{M}$ is generated, according to (11), and let $\{\mu(i)\}_{i=0}^{N-1}$ be an arbitrary PM signal. Then the family of M sets of PM signals

$$\begin{bmatrix} h_{m1}g_{11}\{\mu(i)\}_{i=0}^{N-1} \odot h_{m2}g_{12}\{\mu(i)\}_{i=0}^{N-1} \odot \dots \odot h_{mM}\, g_{1M}\{\mu(i)\}_{i=0}^{N-1} \\ h_{m1}g_{21}\{\mu(i)\}_{i=0}^{N-1} \odot h_{m2}g_{22}\{\mu(i)\}_{i=0}^{N-1} \odot \dots \odot h_{mM}\, g_{2M}\{\mu(i)\}_{i=0}^{N-1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ h_{m1}g_{M1}\{\mu(i)\}_{i=0}^{N-1} \odot h_{m2}g_{M2}\{\mu(i)\}_{i=0}^{N-1} \odot \dots \odot h_{mM}\, g_{MM}\{\mu(i)\}_{i=0}^{N-1} \end{bmatrix}, \tag{12}$$

where $m = 1, 2, \dots, M$ and the symbol "$\odot$" means concatenation, possesses optimal correlation properties.

Proof: According to (5), the generating function (the Hall polynomial), associated with the set of PM signals (12), is

$$F_m(x) = \sum_{k=1}^{M} F_\mu(x) \begin{bmatrix} h_{mk}\, g_{1k} \\ h_{mk}\, g_{2k} \\ \dots\dots \\ h_{mk}\, g_{Mk} \end{bmatrix} x^{(k-1)N} = F_\mu(x)\left[\sum_{k=1}^{M} h_{mk}\, C_{gk}\, x^{(k-1)N}\right], \tag{13}$$

where $F_\mu(x)$ and C_{gk} are defined by (5) and (9) respectively.

After taking in consideration (1) and (13), it can be concluded that the CCF of the m-th and n-th sets of PM signals (12) is associated with the following generating function

$$\begin{gathered} F_m(x)F_n^*(x^{-1}) = F_\mu(x)\left[\sum_{k=1}^{M} h_{mk}\, C_{gk}\, x^{(k-1)N}\right] \times \\ \times F_\mu^*(x^{-1})\left[\sum_{k=1}^{M} h_{nk}^*\, C_{gk}^*\, x^{-(k-1)N}\right] = F_\mu(x)F_\mu^*(x^{-1}) \sum_{r=-M+1}^{M-1} P_{mn}(r)x^{rN} \end{gathered} \tag{14}$$

In (14) $P_{mn}(r), r = -M+1, -M+2, \dots, -1, 0\ ,\ 1, 2, \dots, M-1$ are the samples of the aggregated CCF of rows of m-th and n-th matrices (11), i.e.:

$$P_{mn}(r) = \begin{cases} \sum_{k=1}^{M-\lceil r \rceil} h_{m(k+\lceil r \rceil)} h_{nk}^*\, C_{g(k+\lceil r \rceil)} \otimes C_{gk}^*, \\ \sum_{k=1}^{M-r} h_{mk}\, h_{n(k+r)}^* C_{gk} \otimes C_{g(k+r).}^* \end{cases} \tag{15}$$

In (15) the first row treats the case $-M+1 \le r \le 0$, and the second row treats the case $0 < r \le M-1$.

Now it should be seen, that all products in (15)

$$\begin{gathered} \left[h_{mk}h_{n(k+r)}^*\right]\left[C_{gk} \otimes C_{g(k+r)}^*\right] = 0, \\ \left[h_{mk}h_{n(k+r)}^*\right]\left[C_{gk} \otimes C_{g(k+r)}^*\right] = 0, \end{gathered} \tag{17}$$

are zeros as $C_{gk} \otimes C_{g(k+r)}^* = 0, C_{gk} \otimes C_{g(k+r)}^* = 0$ for the cases $r \neq 0 \cap m \neq n$, according to (9).

When $r = 0$ (15) becomes the form

$$P_{mn}(r) = \sum_{k=1}^{M} h_{mk}h_{nk}^*\, C_{gk} \otimes C_{gk}^* = M\sum_{k=1}^{M} h_{mk}h_{nk}^* = \begin{cases} 0, m \neq n, \\ M^2, m = n, \end{cases} \tag{18}$$

because

$$\sum_{k=1}^{M} h_{mk} h_{nk}^{*} = \begin{cases} 0, m \neq n, \\ M, m = n, \end{cases} \quad (19)$$

according to (10) (more precisely, an analogue to relation (10) is valid for the rows of the unitary orthogonal matrix H).

From (17) and (18) the following conclusions can be made.

C 1) The m-th and n-th radar sensor, which exploit m-th and n-th sets of PM signals (12) by the means of a common system of carrier frequencies

$$\{f_1, f_2, \dots, f_M\} \quad (20)$$

can work simultaneously (i.e. to use simultaneously the system of carrier frequencies (16) for transmission/reception of PM signals, put in the rows of the sets (12)) without any MAIs.

C 2) The aggregated ACF of every set (12) of PM signals is defined by the generating function

$$F_m(x)F_m^*(x^{-1}) = M^2 F_\mu(x)F_\mu^*(x^{-1}). \quad (14)$$

Here, according to (1), $F_\mu(x)F_\mu^*(x^{-1})$ is the generating function, associated with the ACF of the PM signal (2). Due to this reason, in the aggregated ACF of every set (12) of PM signals only $2N-1$ side-lobes can be non-zero, whereas all others $(2MN-1)-(2N-1)$ side-lobes are zeros.

The above conclusions prove the proposition.

4. CONCLUSIONS

In the paper a method for synthesis of families of sets of phase manipulated signals with optimal correlation properties is substantiated. The method allow beginning with a practically arbitrary PM signal derivative wideband PM signals with arbitrary large lengths, very high energy and very low spectral density to be created recursively.

The method could be used in the development of new radar sensor networks, which exploit effectively the electromagnetic spectrum and can operate practically without self-interferences and multi-access interferences.

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REFERENCES:

[1] T. Ignatova, M. Nedelchev, M. Bedzheva, *Contemporary approaches for effective radio spectrum utilization*, Proceedings of University of Ruse - 2022, volume 61, book 3.2., pp. 161-166 (in Bulgarian), ISSN 1311-3321 (print), ISSN 2535-1028 (CD-ROM), ISSN 2603-4123 (on-line);

[2] M. Iliev, M. Bedzheva, *An approach for application of UAVs for observation of processes in agriculture*, Proceedings of 7th International Conference on Energy Efficiency and Agricultural Engineering (EE&AE 2020), 12-14 November 2020, Ruse, Bulgaria;

[3] V. Vasilev, *Stochastic model of complex ISAR signals and spatial correlation image reconstruction procedure*, Proceedings of the 32nd International scientific conference “Modern technologies in the 21st century”, Buharest, Romania, 2007, ISBN 978-973-640-127-5;

[4] P. Kumari, J. Choi, N. Prelcic, R. Heath, IEEE802.11 ad-based radar: An approach to joint vehicular communication-radar system, IEEE Transactions on Vehicular Technology, Volume 67, Issue 4, pp. 3012-3027, April 2018;
[5] B. Bedzhev, D. Dimitrov, Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals, (in press);
[6] B. Bedzhev, D. Dimitrov, A Model of Signal Processing in Software Defined Radio Communication Systems, (in press).