SYNTHESIS AND STUDY OF THE MATHEMATICAL MODEL OF A TRICOPTER

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Abstract: *In this report is synthesized and studied the mathematical model of a tricopter. The stabilization of a tricopter in angular velocity and angle is presented.*

Keywords: *tricopter, mathematical model, stabilization*

1. INTRODUCTION

Regardless of the various types of classification of unmanned aerial vehicles which we have been witness to in the last years, they are divided into two basic kinds – airplanes and helicopters. The helicopter class incorporates the so-called copters.

The application of copters has so far been limited by the capacity of their batteries and by the payload of the different configurations.

One useful configuration is that of the tricopter, which can be seen as quite exotic in terms of steering. For the purposes of this research the cheapest possible design has been selected, where all the three rotors rotate in the same direction. This design features the use of a tilting tail rotor to counterbalance the summary reactive moment of the whole tricopter. The construction schematics of the tricopter that has been developed is shown on Fig. 1.

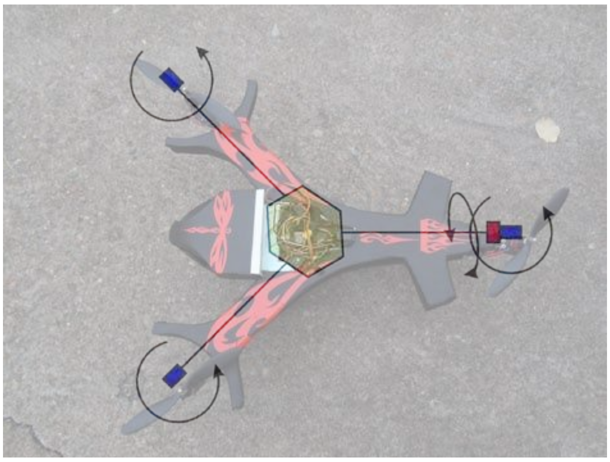

FIG. 1. Construction schematics of the tricopter

The development of computer technology in its two main spheres – hardware and software, made it possible for a large array of new technologies to be implemented in the process of developing and using unmanned aerial vehicles.

Many of these technologies have originated from various universities, for example the Arduino series of controllers and Raspberry Pi class of computers.

Their software base has been developed using primarily Open Source platforms, which makes them easily accessible when designing unmanned aerial vehicles' automatic control systems.

There is a rule regarding the unmanned aerial vehicles' automatic control systems which states that the closer the mathematical model incorporated in the control algorithms is to reality, the more precise the flight control will be.

Following that rule a mathematical model of a tricopter based on a physical model of one such aircraft is presented and studied in the current paper. Stabilization along the axes of the tricopter's body-fixed coordinate system based on the angular velocities and the angle of deflection of the tail rotor is viewed. Fig. 2 depicts the tricopter in flight.

FIG. 2. The studied tricopter in flight

2. SINTHESIZING A COMPLETE MATHEMATICAL MODEL OF A TRICOPTER

The process of synthesizing a working mathematical model of a tricopter passes through several stages. First, the limitations which have to be considered while developing the model are defined. A tricopter is defined as a rigid body with 6 DOF. Thus, the Euler differential equations for quantity of movement and kinetic moment alteration are used for the development of the mathematical model of the tricopter [3]:

$$m.\frac{d\vec{V}}{dt} = \sum \vec{F} \tag{1}$$

where: $\frac{d\vec{V}}{dt}$ is the absolute acceleration in a stationary coordinate system;

$\sum \vec{F}$ is the sum of the external forces acting on the tricopter.

$$\frac{d\vec{K}}{d\,t} = \sum \vec{M} \tag{2}$$

where: $\frac{d\vec{K}}{dt}$ is the kinetic moment; $\sum\vec{M}$ is the summary moment along the axes of a body-fixed coordinate system.

Through equations (1) and (2) is described the forward and rotary motion of a tricopter. In Fig. 3 is shown the distribution of the external forces and moments acting on a tricopter.

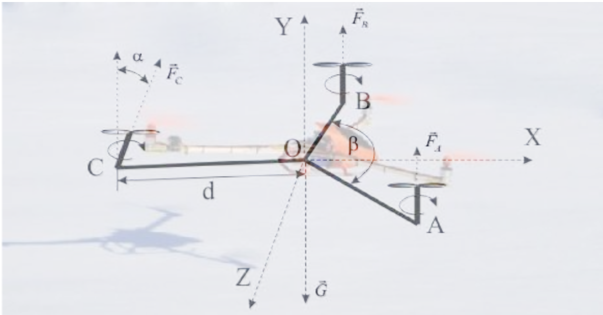

FIG. 3. Diagram of the forces and moments acting on the tricopter

Projecting the external forces from equation (1) along the axes X, Y, Z of the tricopter's body-fixed coordinate system the equations in (3) are worked out, which define the forward movement of the vehicle.

$$\begin{aligned} &\sum F_x = -G.\sin(\ ); \\ &\sum F_y = F_A + F_B + F_C.\cos(\alpha) - G.\cos(\gamma).\cos(\ ); \\ &\sum F_z = F_C.\sin(\alpha) + G.\sin(\gamma).\cos(\ ). \end{aligned} \tag{3}$$

where: F_A, F_B, F_C is the thrust of the engine-propeller system for each of the tricopter's arms; ϑ, γ, ψ are the angles of roll, pitch and yaw; α is the angle of deflection of the tail rotor.

Each of the forces or a combination of the three forces of thrust generated by the tricopter's engines define moments along the axes of the fixed body coordinate system.

Moreover, the propellers of the tricopter generate reactive moments due to the reaction of the air to their rotation.

In reference to Fig. 3, in the equations of the moments along the axes X, Y, Z of the fixed body coordinate system we have the correlations (4):

$$\begin{aligned} &\sum M_x = (F_A - F_B).\sin\left(\beta/2\right).d; \\ &\sum M_y = F_C.\sin(\alpha).d - \left(M_{A_{yp}} f(F_A) + M_{B_{yp}} f(F_B) + M_{C_{yp}} f(F_C, \cos(\alpha))\right); \\ &\sum M_z = (F_A + F_B).d.\cos\left(\beta/2\right) - F_C.\cos(\alpha).d + M_{C_{zp}} f(F_C.\sin(\alpha)). \end{aligned} \tag{4}$$

where: $M_{A_{yp}}, M_{B_{yp}}, M_{C_{yp}}, M_{C_{zp}}$ are the reactive moments generated by the propellers and projected along the axes of the fixed body coordinate system; β is the angle between the arms of the tricopter; d is the length of the tricopter's arm.

The differential equations of the movement of the tricopter are worked out by projecting equations (1) and (2) along the axes of the fixed body coordinate system, which give us the linear and angular velocities in this coordinate system. The forces and moments form equations (3) and (4) constitute the right side of the equations.

The three equations of Euler defining the relation between the angular velocities and the angles of roll (γ), pitch (υ) and yaw(ψ) are used to determine the position in space of the tricopter in relation to a stationary coordinate system. The differential equations are as follows (5):

$$\begin{aligned}
\frac{d\vartheta}{dt} &= \omega_y.\sin\gamma + \omega_z.\cos\gamma \\
\frac{d\gamma}{dt} &= \omega_x - tg\vartheta.(\omega_y.\cos\gamma - \omega_z.\sin\gamma) \\
\frac{d\psi}{dt} &= \frac{\omega_y.\cos\gamma - \omega_z.\sin\gamma}{\cos\vartheta}
\end{aligned} \tag{5}$$

To solve the equations for the movement of the tricopter, for the forces of thrust (F_A, F_B, F_C) and the reactive moments $(M_{A_p}, M_{B_p}, M_{C_p})$ are used the results obtained in (1).

A system of nine differential equations describing the model of the position in space of the tricopter is obtained after the substitution of expressions (3) and (4) in equations (1) and (2) and the addition of the equations from (5) to the system. This is the system of equations (6).

The model of the tricopter's position in space presented with these equations is designed as an S function of a Simulink diagram in a MATLAB-SIMULINK environment.

What is convenient about this method is that it extends Simulink capabilities through the introduction of an additional non-standard block which can be created using Matlab, C, C++ or Fortran. Next, the function is compiled in a MEX file and is dynamically linked with MATLAB execution engine, thus, enabling the S function to be automatically loaded and executed.

$$\begin{aligned}
&m.\left(\frac{dV_x}{dt} + \omega_y.V_z - \omega_z.V_y\right) = -G.\sin \\
&m.\left(\frac{dV_y}{dt} + \omega_z.V_x - \omega_x.V_z\right) = F_A + F_B + F_C.\cos\alpha - G.\cos\gamma.\cos \\
&m.\left(\frac{dV_z}{dt} + \omega_x.V_y - \omega_y.V_x\right) = F_C.\sin\alpha + G.\sin\gamma.\cos \\
&I_x.\frac{d\omega_x}{dt} + (I_z - I_y).\omega_y.\omega_z = (F_A - F_B).d.\sin\left(\frac{\beta}{2}\right) \\
&I_y.\frac{d\omega_y}{dt} + (I_x - I_z).\omega_z.\omega_x = F_C.\sin\alpha.d - \left(M_{A_{yp}}.f(F_A) + M_{B_{yp}}.f(F_B) + M_{C_{yp}}.f(F_C.\cos\alpha)\right) \\
&I_z.\frac{d\omega_z}{dt} + (I_y - I_x).\omega_x.\omega_y = (F_A + F_B).d.\cos\left(\frac{\beta}{2}\right) - F_C.\cos\alpha.d + M_{C_{zp}}.f(F_C.\sin\alpha)
\end{aligned}$$

$$\begin{aligned}
&\frac{d}{dt} = \omega_y.\sin\gamma + \omega_z.\cos\gamma \\
&\frac{d\gamma}{dt} = \omega_x - tg\ .(\omega_y.\cos\gamma - \omega_z.\sin\gamma) \\
&\frac{d\psi}{dt} = \frac{\omega_y.\cos\gamma - \omega_z.\sin\gamma}{\cos}
\end{aligned} \tag{6}$$

In Fig. 4 can be seen the S function of the system of equations (6) presented in a Simuklink diagram

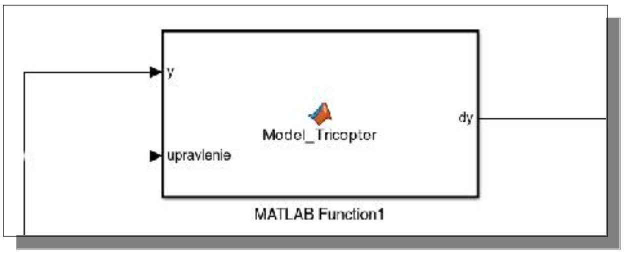

FIG. 4. The S function presented as a simulink diagram

After linearizing the non-linear model, the system's intrinsic numbers equal zero which means that the model is on the border of stability.

In Fig. 5 and Fig. 6 are presented the graphs showing the changes in the angular velocities along the axes of the fixed body coordinate system and the angles of roll, pitch and yaw.

Fig. 7 illustrates the movement of the tricopter with an open-loop control system.

In order to control the tricopter it is necessary for the open-loop system to be closed by means of a feedback connection. The closed-loop system is illustrated in the Simulink diagram in fig. 8[4].

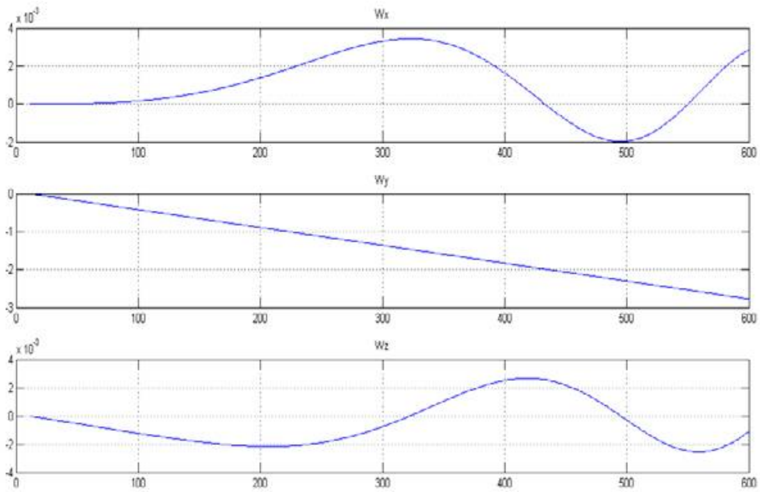

FIG. 5. Changes in the angular velocities of an open-loop system

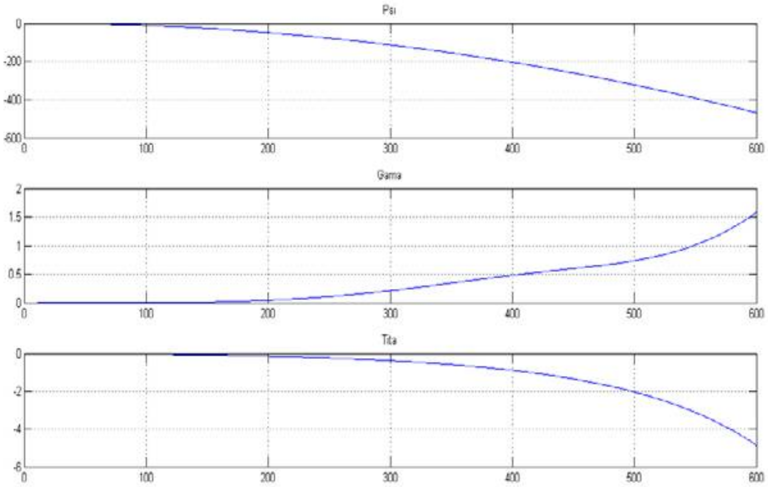

FIG. 6. Changes in roll, pitch and yaw angles of an open-loop system

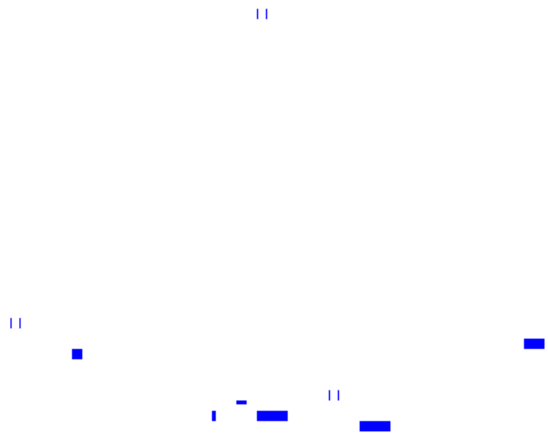

FIG. 7. Spacial movement of an open-loop system

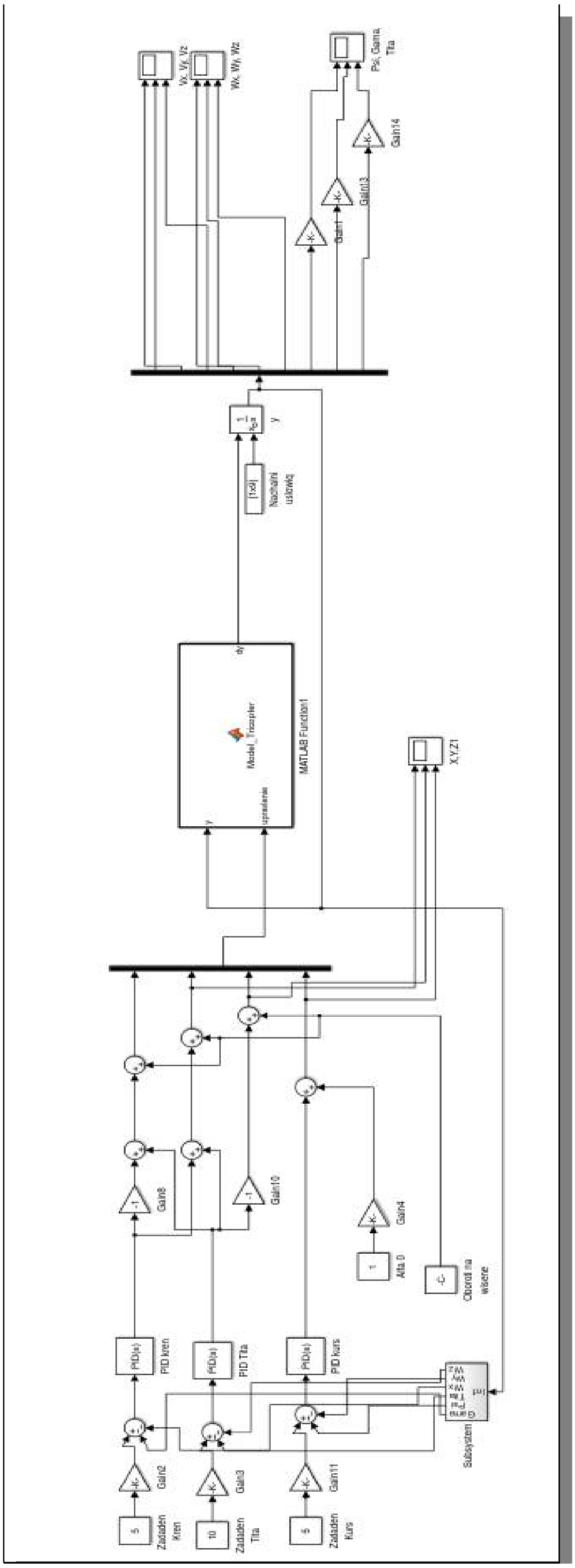

FIG. 8. A simulink diagram of the model with a feedback connection closed-loop system

The case of PID control is studied, using part of the condition vector in the feedback connection [5]. In figures 9 and 10 are shown the changes in the angular velocities along the axes of the fixed body coordinate system and in the roll, pitch and yaw angles, and the picture in fig. 11 shows the tricopter in flight.

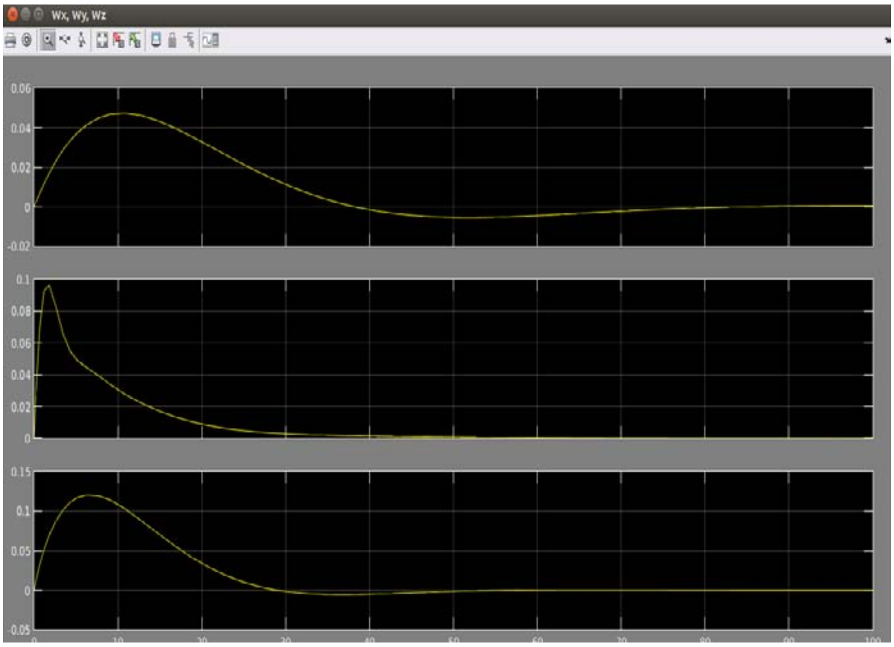

FIG. 9. Changes in the angular velocities of a closed-loop system

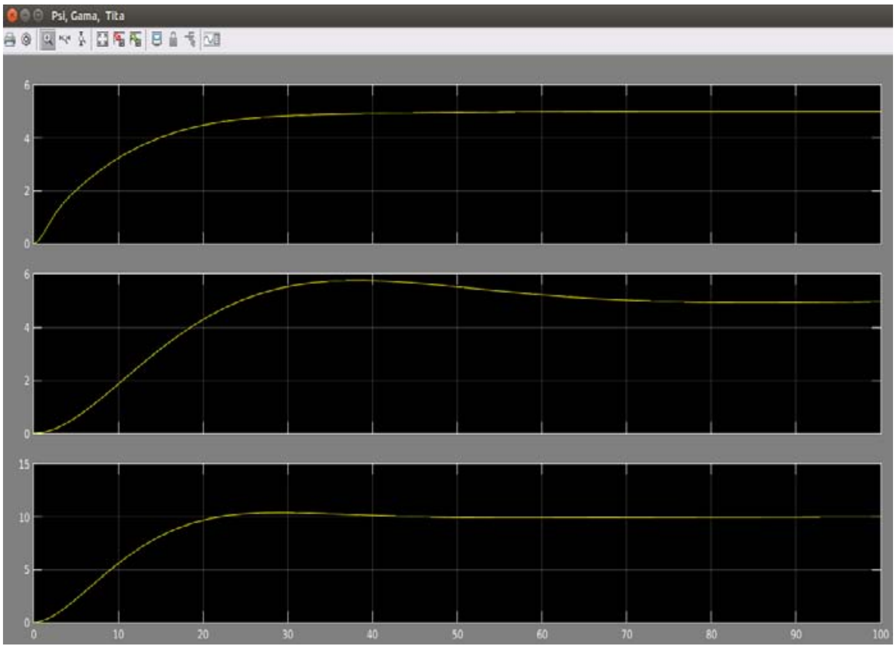

FIG. 10. Changes of roll, pitch and yaw angles of a closed-loop system

FIG. 11. The tricopter in flight

3. CONCLUSIONS AND ACKNOWLEDGEMENTS

In the current paper is developed a mathematical model of the spatial movement of a tricopter.

The open-loop system model is on the border of stability. The closed system model uses PID regulator.

After closing the system it becomes controllable, and the PID regulator's coefficients are automatically adjusted in the MATLAB-Simulink environment.

It can be clearly seen from the results that the PID regulator stabilizes the angular velocities, helping to achieve the predefined angles of divergence.

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