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ALLAN VARIANCE ANALYSIS ON ERROR CHARACTERS OF LOW-COST MEMS ACCELEROMETER MMA8451Q

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Abstract: *This paper gives an evaluation of low-cost MEMS digital accelerometer errors. Such evaluation is required to construct an appropriate model of the accelerometer. Allan Variance is a simple and efficient method for verifying and modelling the errors by representing the root mean square random drift error as a function of averaging time.*

In this paper the characteristics of MEMS accelerometers stochastic errors are identified and quantified, using Allan variance. The derived error model can be applied further to navigation systems of small aircraft.

Keywords: *accelerometer, Allan variance, bias instability, error, random walk, rate random walk.*

1. INTRODUCTION

Advances in the Micro-Electromechanical Systems (MEMS) technology combined with the miniaturization of electronics, have made possible to introduce light-weight, low-cost and low-power chip based inertial sensors for use in measuring of angular velocity and acceleration [9].

MEMS accelerometers output the aircraft acceleration which is used to obtain the position and velocity. The accuracy of accelerometer measurements usually depend on different types of error sources, such as bias, bias instability, velocity random walk, rate random walk, etc. By integrating these measurements in the navigation algorithm, these errors will lead to a significant drift in the position and velocity. To improve the accuracy of aircraft navigation system, different kinds of algorithms are used, integrating accelerometer measurements with other sensors such as GPS receivers [8]. To achieve an accurate estimation of aircraft navigation parameters a good model of sensors' errors is required.

The errors are caused by noise sources which are statically independent, and many approaches for modeling noise are developed. The frequency-domain approach is based on the power spectral density (PSD) [7]. Several time-domain methods have been devised for stochastic modeling [4,14].The simplest and most used is the Allan variance time-domain-analysis technique, originally developed in the mid-1960s to study the frequency stability of precision oscillators [1, 2, 3, 5, 6, 12]. Allan variance is a directly measurable quantity and can provide information on the types and magnitude of various noise terms. The method has been adapted to random-drift characterization of a variety of devices including MEMS accelerometers [15, 16].

In this paper, the Allan variance time-domain-analysis technique is used to characterize the 3-axis, 14-bit/8-bit digital accelerometer MMA8451Q.

2. ALLAN VARIANCE METHOD

If the instantaneous output of the accelerometer is $a(t)$, the cluster average is defined as:

$$\bar{a}_k(\tau)=\frac{1}{\tau}\int_{t_k}^{t_k+\tau}a(t)dt, \quad (1)$$

where $\bar{a}_k(\tau)$ represents the cluster average of the output acceleration for a cluster which starts from the k-th data point. The definition of the subsequent cluster average is:

$$\bar{a}_{k+\tau}(\tau)=\frac{1}{\tau}\int_{t_k+\tau}^{t_k+2\tau}a(t)dt. \quad (2)$$

The difference between the two cluster averages $d_k(\tau)$ is:

$$d_k(\tau)=\bar{a}_{k+\tau}(\tau)-\bar{a}_k(\tau). \quad (3)$$

The variance of differences between every two adjacent cluster averages $\sigma_d^2(\tau)$ is given by:

$$\sigma_d^2(\tau)=\left\langle (d_k(\tau)-\mu_k)^2 \right\rangle, \quad (4)$$

where: μ_k is the mean of $d_k(\tau)$; $\langle\ \rangle$ is the ensemble average.

The Allan variance as function of averaging time is defined as:

$$\sigma^2(\tau)=\frac{\sigma_d^2(\tau)}{2}. \quad (5)$$

Since in digital accelerometers the output data are available in discrete form (5) becomes:

$$\sigma_d^2(nT)=\frac{1}{2(K-1)}\sum_{k=1}^{K-1}(d_k(nT)-\mu_k)^2, \quad (6)$$

where: n is the number of samples in one cluster; K is number of clusters; T is sample time.

It can be shown that the percentage error δ, in estimating the Allan standard deviation of the cluster due to the finiteness of the number of clusters is given by [10]:

$$\delta=\frac{1}{\sqrt{2\left(\frac{N}{n}-1\right)}}, \quad (7)$$

where N is the number of samples in the data set.

To ensure percentage error less than 25% the following expression is obtained from (7):

$$\frac{N}{n}\geq 9 \quad \Rightarrow \quad n\leq\frac{N}{9} \quad (8).$$

Consequently the number of clusters has to be not less than nine.

Different types of random processes cause slopes with different gradients to appear on the log-log plot of Allan standard deviation [11], as shown in Figure 1. Furthermore, different processes usually appear in different regions of τ, allowing their presence to be easily identified [16].

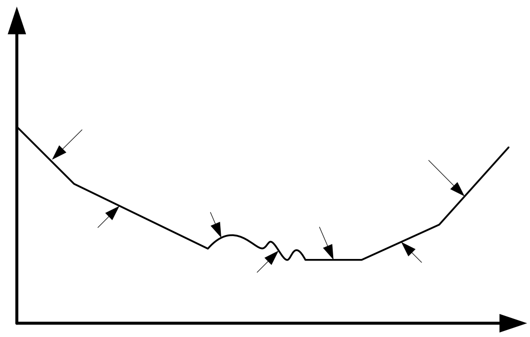

Figure 1. A possible log-log plot of Allan Deviation analysis results

The values of the parameters could be obtained directly from the plot. For a MEMS device such as MMA8451Q the important processes that have to be measured are random walk, bias instability and rate random walk.

The random walk is white noise and appears on the plot with a slope −0.5. The deviation of random walk is obtained by the following expression [11]:

$$\sigma_{rw}=\sigma(\tau_0)\sqrt{\tau_0}, \quad (9)$$

where the plot of $\sigma(\tau)$ has the slope of −0.5 for $\tau=\tau_0$.

The bias instability appears on the plot as a flat region around the minimum. The deviation of bias instability is given by [11]:

$$\sigma_{bi}=\sigma(\tau_1)\sqrt{\frac{\pi}{2\ln(2)}}, \quad (10)$$

where τ_1 is chosen around the minimum of the plot.

The rate random walk is represented by a slope of +0.5 on a log-log plot of $\sigma(\tau)$ and its deviation is given by [11]:

$$\sigma_{rrw} = \sigma(\tau_2)\sqrt{\frac{3}{\tau_2}}, \qquad (11)$$

where the plot of $\sigma(\tau)$ has the slope of +0.5 for $\tau = \tau_2$.

3. RESULTS

The data from MMA8451Q is read using a device described in [13]. The accelerometer is set up to: 14 bits of resolution; ± 2 g full scale; 100 Hz output data rate. The data was collected for eight hours. Collected accelerations for each axis (*x*,*y*,*z*) are shown in figures from 2 to 4.

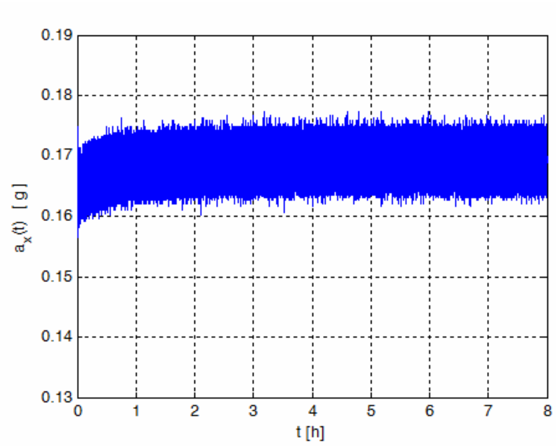

Figure 2. Measured acceleration in *x*-axis.

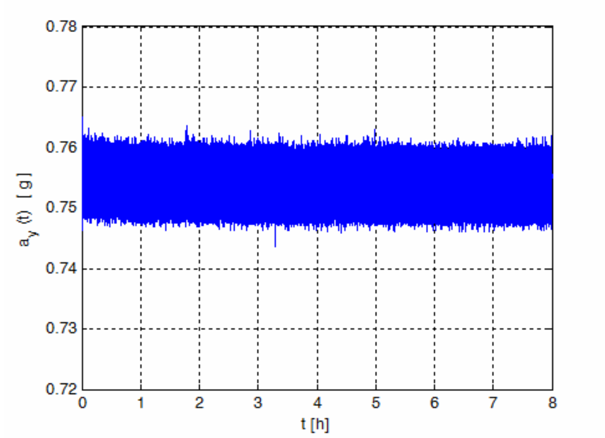

Figure 3. Measured acceleration in *y*-axis.

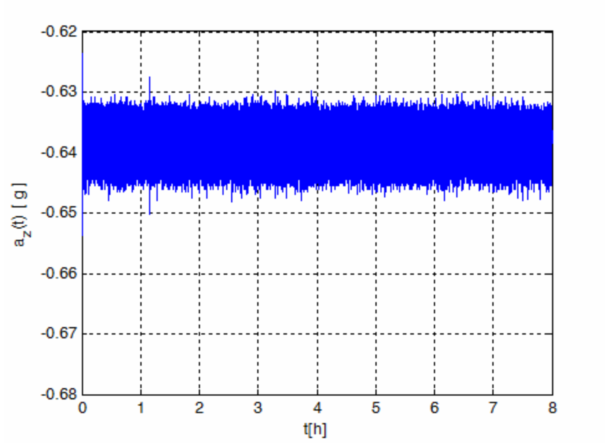

Figure 4. Measured acceleration in *z*-axis.

The orientation of the accelerometer was such that none of the axes was aligned with Earth gravity. The alteration of acceleration in each axis is of different character, consequently the accelerometer errors are different too.

Allan standard deviations for each axis (*x*,*y*,*z*) are presented in figures from 5 to 7.

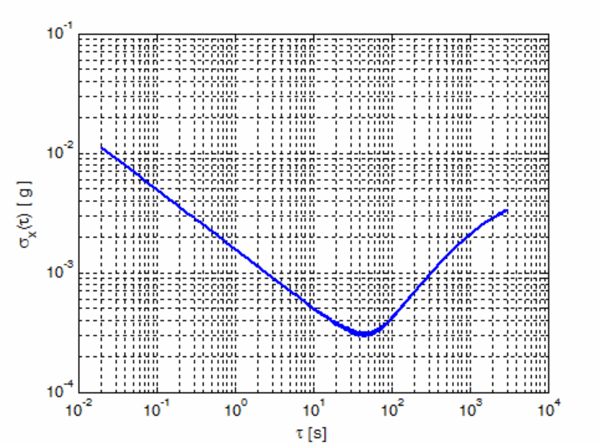

Figure 5. Allan standard deviation for x-axis.

Figure 5 shows that the slope of Allan Deviation curve for $\tau = 0.1 \div 1\,s$ is -0.5002. The value of $\sigma_{rw} = 1.5742\ mg$ is obtained for deviation of random walk, using equation (9) with $\tau_0 = 1s$. For the deviation of bias instability is measured $\sigma_{bi} = 0.46508\ mg$ at $\tau_1 = 45s$. The slope of Allan Deviation curve for $\tau = 70 \div 80\,s$ is +0.4928. Deviation of rate random walk is obtained from (11) at $\tau_2 = 75s$ and it is $\sigma_{rrw} = 0.0692\, mg/\sqrt{s}$.

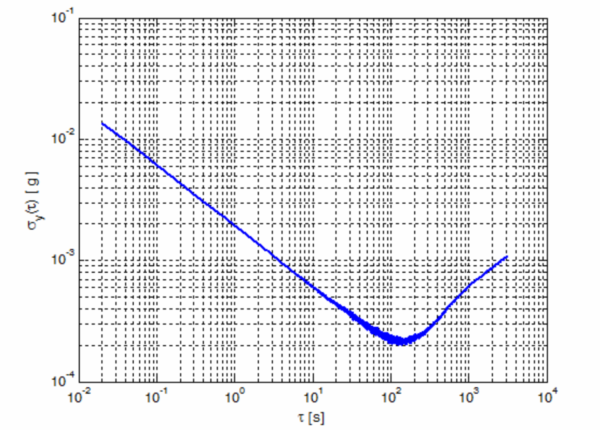

Figure 6 Allan standard deviation for y-axis.

Figure 6 shows that the slope of Allan Deviation curve for $\tau = 0.1 \div 1\,s$ is -0.4978. The value of $\sigma_{rw} = 1.9408\ mg$ is obtained for deviation of random walk, using equation (9) with $\tau_0 = 1s$. For the deviation of bias instability is measured $\sigma_{bi} = 0.3158\ mg$ at $\tau_1 = 142s$. The slope of Allan Deviation curve for $\tau = 280 \div 320\,s$ is +0.4970. Deviation of rate random walk is obtained from (11) at $\tau_2 = 300s$ and it is $\sigma_{rrw} = 0.0930\, mg/\sqrt{s}$.

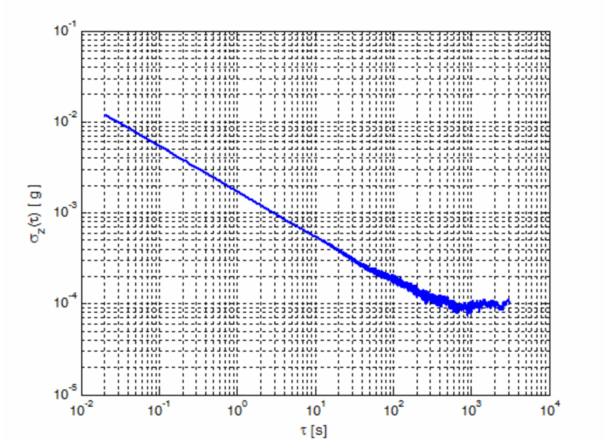

Figure 7. Allan standard deviation for z-axis.

Figure 7 shows that the slope of Allan Deviation curve for $\tau = 0.1 \div 1\,s$ is -0.5019. The value of $\sigma_{rw} = 1.7118\ mg$ is obtained for deviation of random walk, using equation (9) with $\tau_0 = 1s$. For the deviation of bias instability is measured $\sigma_{bi} = 0.1266\ mg$ at $\tau_1 = 940s$. There isn't slope of Allan Deviation curve for obtaining the rate random walk.

According to MMA8451Q technical data the square root of power spectral density is $126\ \frac{\mu g}{\sqrt{Hz}}$ for output data rate (ODR) of 400 Hz. The minimum and maximum value of output data bandwidth is $\Delta f_{\min} = \frac{ODR}{3}$ and $\Delta f_{\max} = \frac{ODR}{2}$. Using these parameters a following range of random walk is obtained $\sigma_{rw} = 1.4549 \div 1.7819$ mg.

The relative Earth acceleration is calculated, using the following equation:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}\ . \qquad (12)$$

The calculated relative Earth acceleration is shown on figure 8.

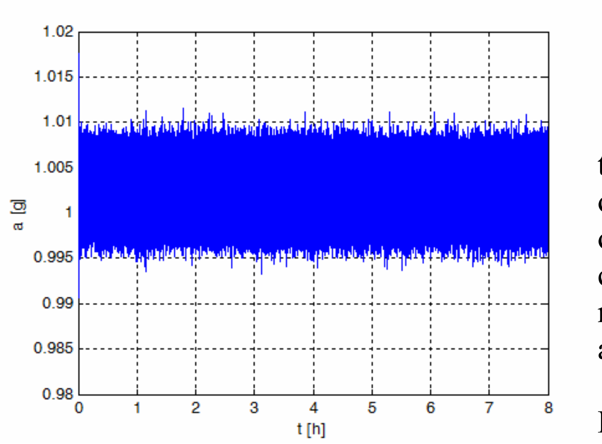

Figure 8. Relative Earth acceleration.

Allan standard deviations for measured Earth acceleration is presented in figure 9.

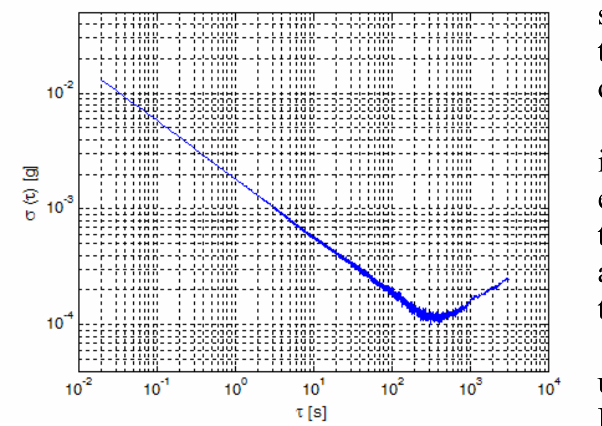

Figure 9. Allan standard deviation for measured Earth acceleration.

Figure 9 shows that the slope of Allan Deviation curve for $\tau = 0.1 \div 1\,s$ is -0.4991. The value of $\sigma_{rw} = 1.8390\ mg$ is obtained for deviation of random walk, using equation (9) with $\tau_0 = 1s$. For the deviation of bias instability is measured $\sigma_{bi} = 0.1746\ mg$ at $\tau_1 = 400s$.

4. CONCLUSIONS

The Allan-variance technique presented in this paper allows a systematic characterization of the various random errors in the output data of the digital accelerometer. The characteristic curves are obtained and different types and magnitude of error terms existing in the accelerometer MMA8451Q are determined.

The eight-hour static data from the MMA8451Q were investigated. Most of the results from the Allan-variance analysis are close to the manufacturers' claim, which proves that the method presented in this paper is valid.

There isn't a slope of -1 on plot of Allan standard deviations (Fig.5-7). Consequently the quantization errors are much less than other errors and can be ignored.

The results of the MMA8451Q clearly indicate that the random walk is the dominant error term in the short cluster time, whereas the bias instability and rate random walk terms are the dominant errors in the long cluster time.

The experimental results have provided a useful evaluation of a low-cost accelerometer MMA8451Q.

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