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CONTROLLED FLOW SIMULATION USING SPH METHOD

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Abstract: *Smoothed Particle Hydrodynamics (SPH) method is a meshless numerical method, developed after 1977 and nowadays this method is practically validated as fluid flow is concerned. The most important aspects and the most difficult problems, like fluid free surface, interaction fluid-structure any many others can be better solved and in a shorter time too. This paper presents some theoretical fundamentals of the SPH method, including specific approximation of Navier-Stokes equations in this numerical method and next to it, an interesting application is also presented. The application is referring to the numerical simulation of a controlled flow, for instance, through a gate. This fluid flow numerical simulation, in such conditions, has many practically applications and is an implicit describing of the free fluid surface. The authors hope that this numerical method (SPH) to be a fitted one, in applications such aerodynamics and a direct analysis of structure behavior under aerodynamic loads. These aspects are over the subject of this paper, but in future, we will present more details and examples regarding using of the SPH method in aerodynamics. We also hope that this paper to be useful for teachers, researchers and students interested in SPH method.*

Keywords: *SPH, fluid flow, numerical analysis, kernel function, smoothing length*

1. INTRODUCTION

SPH (Smoothed Particle Hydrodynamics) method is a griddles Lagrangian technique which comes from astrophysics (Lucy, 1977). The method was extended to fluid simulation, especially with free-surface (Monaghan, 1992), nowadays SPH method being also used in many scientific fields. Applied mechanics domain is perhaps the last one, but it is intensively researched and significant advances have also been made.

2. THEORETICAL FUNDAMENTALS OF THE SPH METHOD

The SPH method belongs to the meshless methods, so the investigated domain is represented by a number of nodes, representing the particles of this domain, having their material and mechanical (mass, position, velocity etc.) characteristics. Each particle represents an interpolation point on which the material properties are known.

Practically, depanding on the used software, the boundary conditions have to be imposed to some of particles, according to the problem analyzed, like in the case of finite element method. Theoretical fundamentals of this aspect are a bit different and require more space to be presented here.

The problem solution is given by the computed results, on all the particles, using an interpolation function. We can say that the fundamentals of SPH theory consist in interpolation theory; all the behavior laws are transformed into integral equations.

The kernel function, or smoothing function, often called smoothing kernel function, or simply kernel, gives a weighted approximation of the field variable (function) in a point (particle). Integral representation of a function $f(x)$, used in the SPH method starts from the following identity:

$$f(x) = \int_{\Omega} f(x')\delta(x - x')dx' \qquad (1)$$

where f is a function of a position vector x, which can be an one-, two- or three-dimensional one; $\delta(x - x')$ is a Dirac function, having the properties:

$$\delta(x - x') = \begin{cases} 1 \rightarrow x = x' \\ 0 \rightarrow x \neq x' \end{cases} \qquad (2)$$

In equation (1), Ω is the function domain, which can be a volume, that contains the x, and where $f(x)$ is defined and continuous. By replacing the Dirac function with a smoothing function $W(x - x', h)$ the integral representation of $f(x)$ becomes:

$$f(x) = \int_{\Omega} f(x')W(x - x', h)dx' \qquad (3)$$

where W is the smoothing kernel function, or smoothing function, or kernel function. The parameter h, of the smoothing function W, is the smoothing length, by which the influence area (support domain) of the smoothing function W is defined.

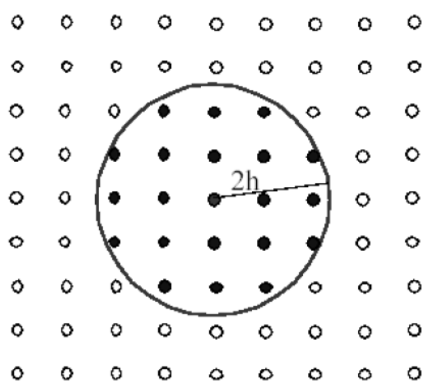


Fig. 1 The support domain of the kernel function

As long as Dirac delta function is used, the integral representation, described by equation (1), is an exact (rigorous) one, but using the smoothing function W instead of Dirac function, the integral representation can only be an approximation. This is the reason for the name of kernel approximation. Using the angle bracket, this aspect is underlined and the equation (3) can be rewritten as:

$$\langle f(x) \rangle = \int_{\Omega} f(x')W(x - x', h)dx' \qquad (4)$$

The smoothing function W is usually chosen to be an even one, which has to satisfy some conditions. The most important requirements of a kernel function are presented below:

a) the smoothing function has to be *normalized* over its support:

$$\int_{\Omega} W(x - x', h)dx' = 1 \qquad (5)$$

b) the smoothing function has to be *compactly supported:*

$$W(x - x', h) = 0 \text{ for } |x - x'| > kh \qquad (6)$$

c) the smoothing function has to be *positive* for any point at x' within the support domain:

$$W(x - x', h) \geq 0 \qquad (7)$$

d) the smoothing function value has to be *monotonically decreasing* with the increase of the distance away from the particle.

e) the smoothing function value has to satisfy the *Dirac delta function condition* as the smoothing length approaches to zero:

$$\lim_{h \to 0} W(x - x', h) = \delta(x - x') \qquad (8)$$

f) the smoothing function value has to be an *even function* (symetric).

The literature presents different smoothing function. Theoretically, any function having the properties presented above, can be employed as SPH smoothing function.

One of the most used smoothing function is a cubic B-spline kernel function, in the form given by relation (9), where *s = r/h, n* is the number representing the spatial dimension and α is a constant which has the value: 2/3, $10/7\pi$ or respectivelly $1/\pi$, depending on the space dimension (1D, 2D or 3D).

$$W(s,h)=\frac{\alpha}{h^n}\begin{cases}\left(1-\frac{3}{2}s^2+\frac{3}{4}s^3\right);0\le s<1\\ \frac{1}{4}(2-s)^3;1\le s<2\\ 0;s\ge 2\end{cases} \quad (9)$$

The graphical representation of this smoothing function and its derivatives (first and second) can be seen in the Figure 2.

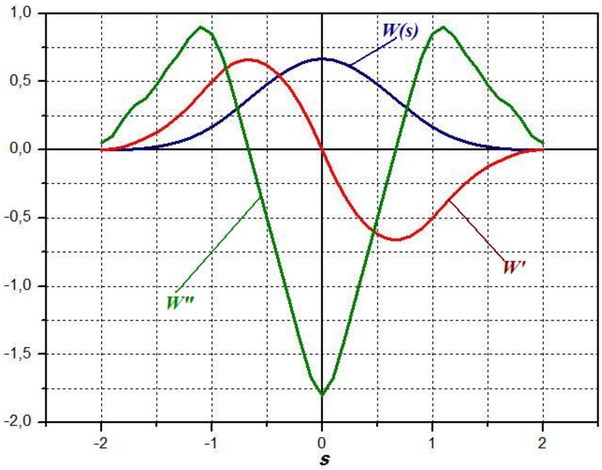


Fig. 2 The cubic B-spline kernel function

3. SPH FORMULATION OF THE NAVIER-STOKES EQUATIONS

The Navier-Stokes equations consist of the following equation set, representing three fundamental physical laws of conservation, which in SPH formulation represents a summation of the Lagrangian form:

a) the continuity ecuation:

$$\frac{\partial \rho_i}{\partial t}=\rho_i\sum_{j=1}^{N}\frac{m_j}{\rho_i}v_{ij}^{\alpha}\frac{\partial W_{ij}}{\partial x_i^{\alpha}} \quad (10)$$

where x_i is a generalized coordinate and α denotes the coordinate directions.

As the density is concerned, this is expressed in the following two forms, the last being proposed for improving the accuracy near the free boundaries or near the material interfaces having a density discontinuity.

$$\rho_i=\sum_{j=1}^{N}m_jW_{ij} \quad (11)$$

$$\rho_i=\frac{\sum_{j=1}^{N}m_jW_{ij}}{\sum_{j=1}^{N}\left(\frac{m_j}{\rho_j}\right)W_{ij}} \quad (12)$$

b) the momentum equation:

$$\frac{\partial v_i^{\alpha}}{\partial t}=\sum_{j=1}^{N}m_j\frac{\sigma_i^{\alpha\beta}+\sigma_j^{\alpha\beta}}{\rho_i\rho_j}\frac{\partial W_{ij}}{\partial x_i^{\beta}} \quad (13)$$

$$\frac{\partial v_i^{\alpha}}{\partial t}=\sum_{j=1}^{N}m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2}+\frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} \quad (14)$$

Both forms are used, the last being more popular. The symmetrized form of both equations leads to reducing of the errors.

c) the energy equation:

$$\frac{de_i}{dt} = \frac{1}{2}\sum_{j=1}^{N} m_j \frac{p_i + p_j}{\rho_i \rho_j} v_{ij}^{\beta} \frac{\partial W_{ij}}{\partial x_i^{\beta}} + \frac{\mu_i}{2\rho_i} \varepsilon_i^{\alpha\beta} \varepsilon_i^{\alpha\beta} \qquad (15)$$

$$\frac{de_i}{dt} = \frac{1}{2}\sum_{j=1}^{N} m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) v_{ij}^{\beta} \frac{\partial W_{ij}}{\partial x_i^{\beta}} + \frac{\mu_i}{2\rho_i} \varepsilon_i^{\alpha\beta} \varepsilon_i^{\alpha\beta} \qquad (16)$$

The above forms are most used in many SPH software. The superscripts α and β, used in the relations (13)…(16) are used for denoting the coordinate directions. The others notations are those frequently used in fluid mechanics (ε is shear strain rate, μ is the dynamic viscosity and p is the pressure).

4. CONTROLLED FLOW SIMULATION USING SPH METHOD

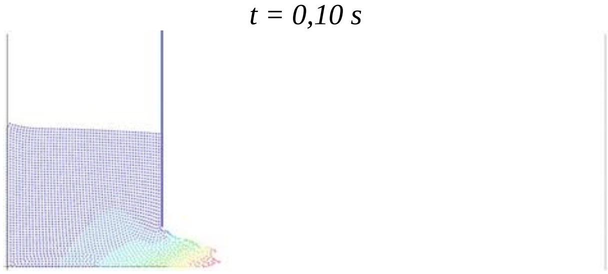
The controlled flow is referring to the collapse of a water quantity ($1\,m^3$) through a gate with a variable space by a law. The model is a 2D one and its original state is presented in the Figure 3.

Fig. 3 The 2D SPH model

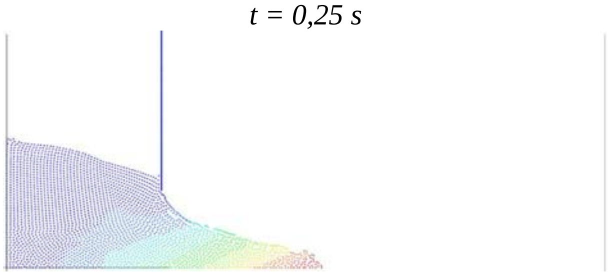
The water is modelled by 2601 particles with a nodal distance of 2 cm. Only the weight as body force is considered.

The gate moving on vertical direction and its coming back in a time shorter than analysis period, the water will be in two domains, at different levels. Figure 4 presents the water evolution at different moments. An elastic-plastic-hydro material model was used with the water charateristic parameters.

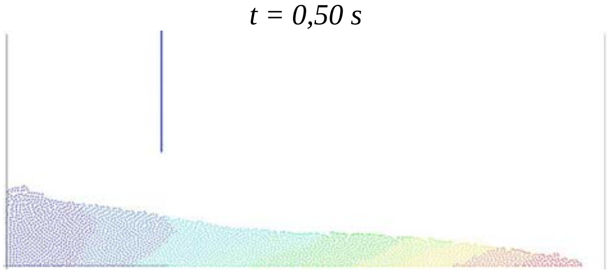
t = 0,10 s



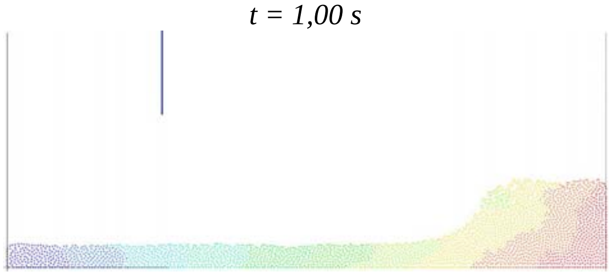
t = 0,25 s



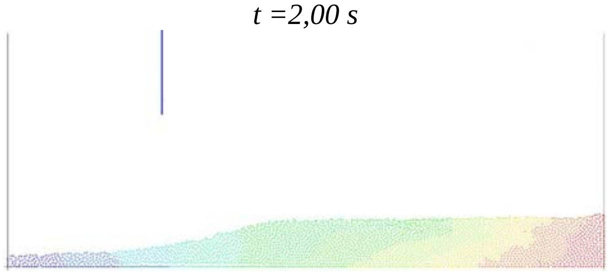
t = 0,50 s



t = 1,00 s



t =2,00 s



t =3,00 s

t =4,00 s

t =5,00 s

t =6,00 s

Fig. 4 Fluid evolution in time

Figure 4 presents, in the same time, the fluid state and the velocity field of particles.

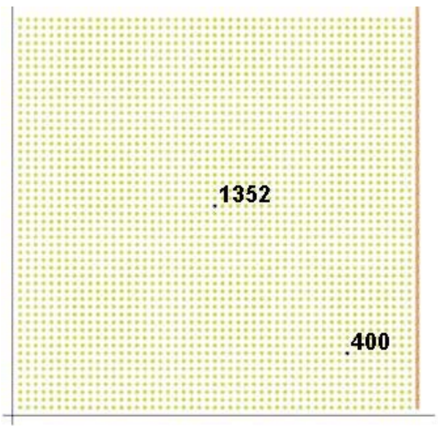


Fig. 5 Initial position of two particles

In the figures 6…9 the time evolution of two particles is presented. The initial position of these particles is shown in the Figure 5.

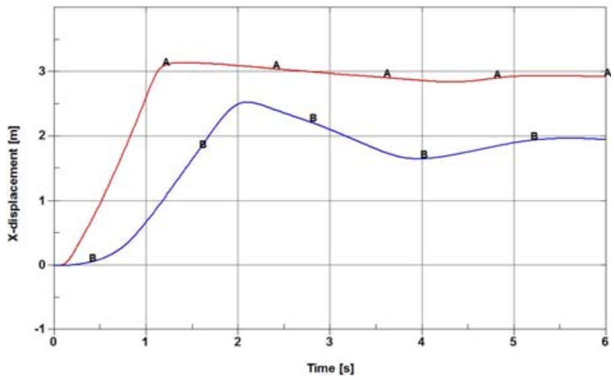


Fig. 6 UX displacement of those two particles

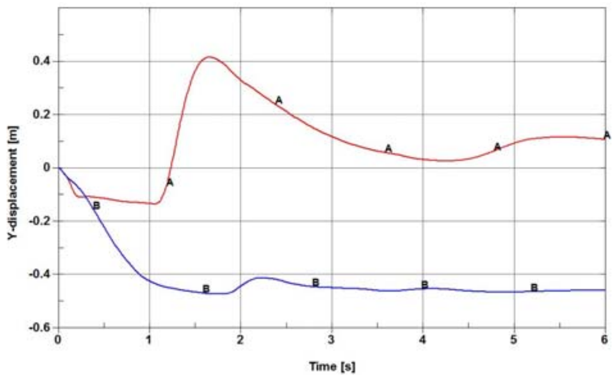


Fig. 7 UY displacement of those two particles

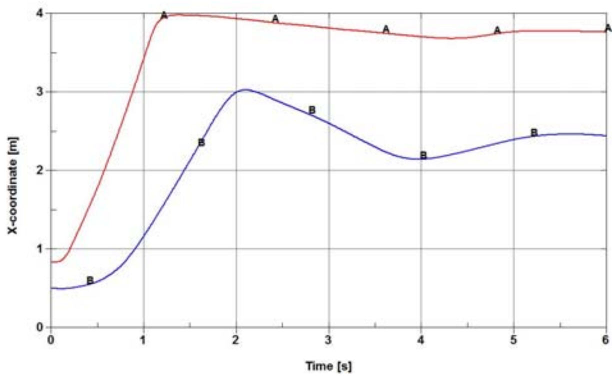


Fig. 8 X-coordinate of those two particles

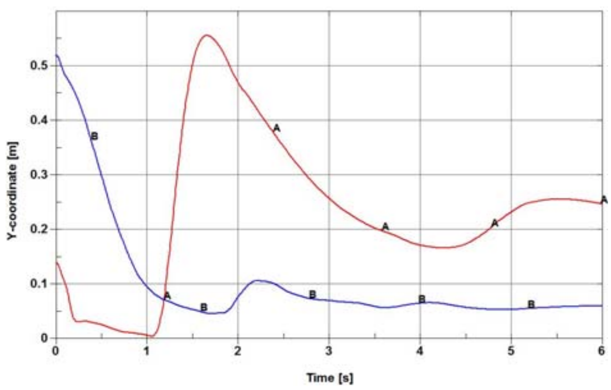


Fig. 9 Y-coordinate of those two particles

In the figures 6…9, particle A is the particle 400 and particle B is the particle 1352, presented in the Figure 5.

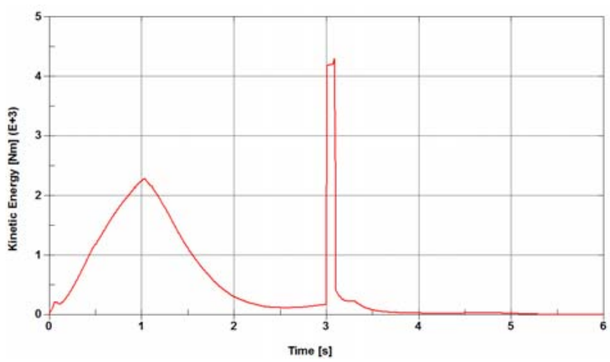


Fig. 10 The evolution in time of the water kinetic energy

The water kinetic energy, presented in the Figure 10 is in connection with the moving law of the gate, presented in the Figure 11, as everyone could notice and understand the curve allure describing the kinetic energy of the water. At the end of the analysis period (six seconds), the kinetic energy is zero, this meaning a properly analysis time.

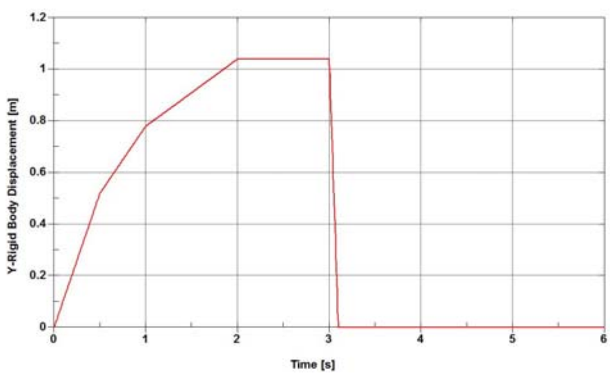


Fig. 11 The moving law of the gate

Analysing the evolution of the X anY coordinates and UX and UY displacements, we can know where those two particles will be finally. So, for this case, the both particles will be in the left side of the gate (figure 4), where the water level is greater than the right side of the gate. We also can obtain, by graphical post-processing of the results, the water level around the gate.

5. CONCLUSIONS

The SPH hydrodynamics model used in this paper can be an example for many other similar problem, SPH method being a very powerful method. The same problem could be solved using a 3D model, but more computer time would have been necesary.

Practically, SPH method is validated as an eficient numerical method in fluid mechaqnics.

Our work is intended as a recommendation for using the SPH method in researching and even in education. Many facilities are going to be discovered and only the method power and the user talent can lead to unexpected results.

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