MATHEMATICAL MODEL FOR AN AIRCRAFT TURBOSHAFT-TYPE AUXILIARY POWER UNIT

Alexandru-Nicolae TUDOSIE

University of Craiova, Craiova, Romania (atudosie@elth.ucv.ro)

DOI: 10.19062/1842-9238.2017.15.3.21

Abstract: *In this paper the author has studied an aircraft auxiliary power unit (APU), identified as controlled object, referring to the specific case of a TG-16M unit (consisting of a gas turbine turbo-shaft which spins up an electrical 28 V DC generator, treated as an embedded system). From APU's behavior point of view, the author has identified embedded system's non-linear motion equations, then, using the finite difference method, the equation system was brought to a linear, then to an adimensional form, which is more appropriate for further studies. System's coefficients were experimentally established or calculated and/or estimated during several ground lab tests of the TG-16M unit, using the facilities of Aerospace Engineering Laboratory (at the University of Craiova). Based on system's determined mathematical model and transfer function, the author has also performed some studies concerning its stability and quality, studies realized using Matlab® Simulink simulations and some conclusions were expressed.*

Keywords: *APU, fuel, rotational speed, gas turbine, electrical power, generator, control.*

1. INTRODUCTION

From aircrafts point of view, an auxiliary power unit (APU) is a small gas turbine engine, which can provide a certain power (electrical power and/or compressed air power) to on-board consumers; it allows them an autonomous operation, without reliance on specific ground support equipment (such as electrical power units/batteries/generators, external air-conditioning units, or any other auxiliary ground equipment) [14].

APU's main goal is to provide power to almost all airplane's essential consumers while it is on the ground (on the runway or platform); in most of cases, APU cannot offer supplementary propulsion, but its generated power is used to: a) start the main propulsion engine(s); b) provide another form(s) of energy for several systems (pressurized air for environmental control system, or electrical power for on-board equipments, avionics, lighting system(s), or else). Additionally, an APU can provide during the flight backup and emergency power (such as supplementary power for deicing system) [3].

An APU may be used as a starter for aircraft's main engines, which needs to be accelerated to rotate at an extremely high speed and kept spinning, in order to assure sufficient air compression.

Depending on its destination, the APU is designed to be able to provide electric, hydraulic, or pneumatic power (even all three of them). The APU connection to a hydraulic pump allows a safe back-up operation of hydraulic equipment (such as the flight controls, aerodynamic brakes or flaps), even if an engine failure occurs. Other APUs are designed only for ground use (engine start, air conditioning), but if certified for use in flight, an APU is also useful for supplementary electrical power (if an engine driven generator fails), or as a source of bleed air for air conditioning.

The APU is normally left off in flight, but, when necessary, it may be turned on as an extra precaution, especially for transoceanic flights, severe conditions flights and/or the icing hazard is present.

APU's are positioned on different locations on the airplane (aircraft). Often them are mounted in the tail (in the rear fuselage, as Fig. 1 shows), or in the rear of engines' nacelles (for example on An-24 or Il-18), as well as in the landing gear bay. As a precaution, the APU is installed in the far aft tail-cone section; it realizes APU's safe isolation from other critical systems (in the unlikely event of a fire, or failure). Consequently, it is compulsory that APUs are installed behind secure firewalls (as aircraft main engines are) and they also need their own fire detection and extinguishing system [3,8].

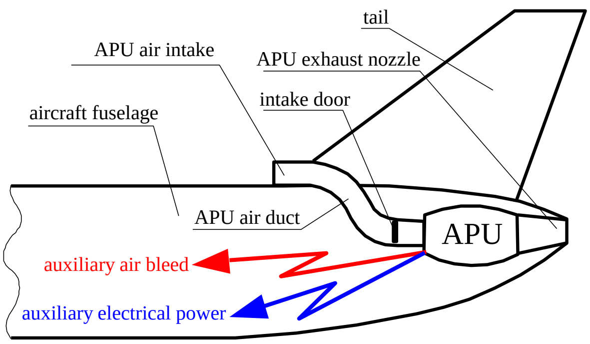

FIG. 1. APU's positioning on aircraft

2. DESCRIPTION OF APU'S ARCHITECTURE AND OPERATION[14]

The TG-16M APU, studied in [14] and [15], is a turboshaft-type (very similar to a turboprop engine, but, instead a propeller, it spins up a dedicated 28 V DC electrical generator). A planetary or a conventional gear is used to reduce the engine's turbine speed to an appropriate value for the generator. It operates as auxiliary electrical power source for an airplane (such as An-24T short-haul passenger airplane, or the old Il-18 medium-haul passenger airplane); it is positioned in the rear of starboard AI-24 engine's nacelle.

The studied APU consists of three main sections, depicted in Fig.2: a) the power main section (the turbo-engine, or the gas generator); b) the planetary or classical gearbox; c) the electrical DC generator. The power section consists of the gas generator portion of the engine and produces all the power for the entire APU (engine's compressor, reduction gear and electrical generator).

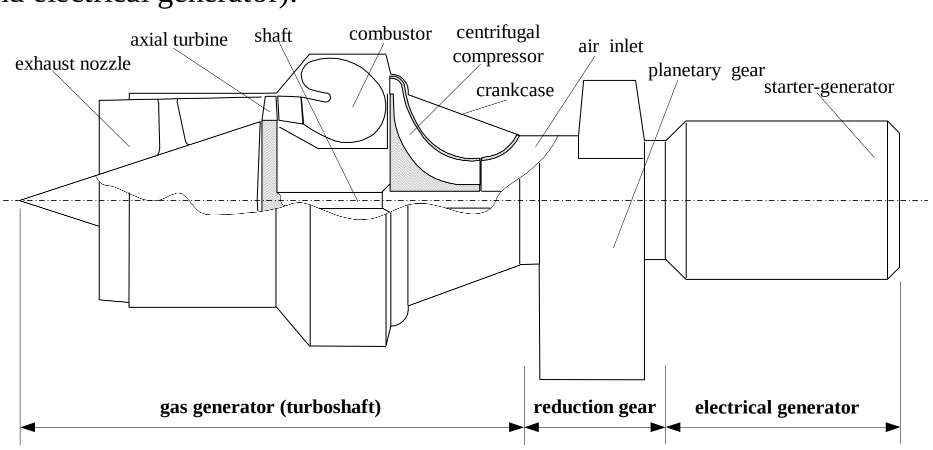

FIG. 2. Schematics of a TG-16M-type APU with its main parts

As Fig. 2 shows, APU's main shaft is driven by a single stage axial turbine, which is the only mechanical work producer. So, it spins up both engine's centrifugal compressor and the electrical generator (via the reduction gear). The engine (the gas generator) has a compact combustor (with inverted flow), where the injected fuel is intimately mixed with the compressed air (which s delivered by the compressor). Burning process transform them into hot gases, which are, firstly, expanded in the engine's turbine (producing mechanical work), then discharged through the exhaust nozzle. Turbine's mechanical work should cover the necessary work of several parts, such as the compressor, the reduction gear and the electrical generator ([4], [7], [8]).

3. EMBEDDED SYSTEM'S NON-LINEAR MOTION EQUATIONS

Non-linear mathematical model, as determined in [14], consists of APU-system main parts' non-linear equations. System main parts, as shown in Fig. 2, are: a) APU's main shaft'; b) rotational speed transducer; c) fuel pump and fuel control system. In Fig. 3 APU's fuel system's technical schematic is depicted.

Shaft motion equation involves a few torques ([1], [12]), as follows:

$$M_T - M_C - M_g - M_{fr} - M_{EG} = \frac{\pi J}{30}\frac{\mathrm{d}n}{\mathrm{d}t}, \tag{1}$$

where M_T is turbine's torque, M_C – compressor's, M_g – gear, M_{fr} – friction, M_{EG} – electrical generator torques, J is spool's axial moment of inertia and n – shaft rotational speed.

APU's fuel system motion equations are:

$$Q_i = Q_p - Q_C, \tag{2}$$

$$Q_C = \mu_d \frac{\pi d_1^2}{4}\sqrt{\frac{2}{\rho}}\sqrt{p_a - p_C}, \tag{3}$$

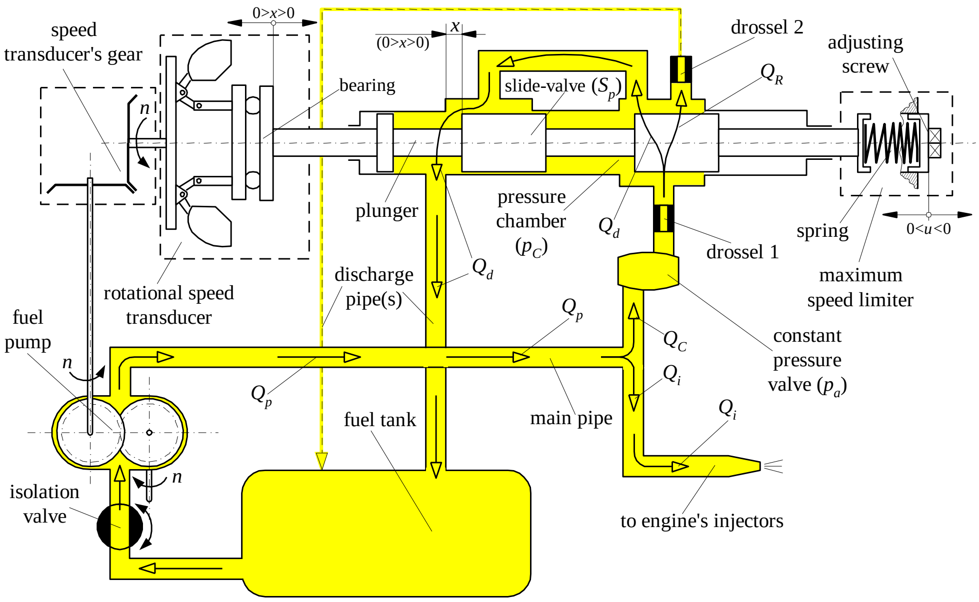

FIG. 3. APU's fuel system constructive and operational diagram

$$Q_R = \mu_d \frac{\pi d_2^2}{4}\sqrt{\frac{2}{\rho}}\sqrt{p_C - p_{dc}}\ , \tag{4}$$

$$Q_d = \mu b x\sqrt{\frac{2}{\rho}}\sqrt{p_C - p_{dc}}\ , \tag{5}$$

$$Q_C - Q_R - Q_d = S_p \frac{\mathrm{d}x}{\mathrm{d}t} + \beta_f V_C \frac{\mathrm{d}p_C}{\mathrm{d}t}\ , \tag{6}$$

where p_a is the fuel supplying pressure (assumed as constant), p_C – command pressure, p_{dc} – discharge pipes fuel pressure (negligible because its very low value), μ, μ_d – flow coefficients, β_f – fuel's compressibility coefficient (assumed as null), S_p – slide-valve plunger's surface area, b – slide-valve's discharge slot width, V_C – pressure chamber's volume, Q_i – injection fuel flow rate (which supplies engine's combustor and it is established as the difference between pump's flow rate Q_p and control flow rate Q_C, recycled and sent back to the fuel tank). In the slide valve's pressure chamber the control flow rate is split into two streams: Q_R – through the second drossel and Q_d – through the variable slot of plunger's slide valve and further through the discharge pipe back into the fuel tank. Consequently, discharged flow rate Q_d depends on the plunger's displacement *x*, which results from engine's effective speed *n* and from preset speed value n_p (given by the adjusting screw's displacement *u*).

4. EMBEDDED SYSTEM'S MATHEMATICAL MODEL AND TRANSFER FUNCTION

Assuming the small perturbation hypothesis, one has used the finite difference method ([12], [13]) in order to bring non–linear equations to a linear form. Thus, any variable or parameter X should be formally considered as $X = X_0 + \Delta X$ (where X_0 is X parameter's steady state value, ΔX – parameter's deviation, while $\overline{X} = \frac{\Delta X}{X_0}$ the non-dimensional deviation). Using some appropriate chosen amplifying terms, linearised equations can be transformed into non-dimensional forms; after applying the Laplace transformation, one obtains system's linear non-dimensional mathematical model, as follows:

$$(\tau_m \mathrm{s} + 1)\overline{n} = k_f \overline{Q}_i - k_{cg}\overline{I}_{cg}\ , \tag{7}$$

$$\overline{Q}_i = \overline{Q}_p - \overline{Q}_C\ , \tag{8}$$

$$\overline{Q}_C = -k_{QC}\overline{p}_C\ , \tag{9}$$

$$\left(\tau_x s+1\right)\bar{x}=\frac{1}{k_{xC}}\,\bar{p}_C\,, \tag{10}$$

where $k_{mR}=\left(\frac{\partial M_C}{\partial n}\right)_0+\left(\frac{\partial M_{EG}}{\partial n}\right)_0-\left(\frac{\partial M_T}{\partial n}\right)_0$, $\tau_m=\frac{\pi J}{30k_{mR}}$, $k_f=\frac{1}{k_{mR}}\frac{Q_{i0}}{n_0}\left(\frac{\partial M_T}{\partial Q_i}\right)_0$,

$\tau_x=\frac{S_p}{k_{dx}}$, $k_{cg}=\frac{1}{k_{mR}}\frac{Q_{i0}}{I_{cg0}}\left(\frac{\partial M_{EG}}{\partial I_{cg}}\right)_0$; $k_{QC}=k_{CC}\frac{Q_{C0}}{p_{C0}}$, $k_{xC}=\frac{x_0}{\left(k_{CC}-k_{RC}-k_{dC}\right)p_{C0}}$.

One has to consider also fuel pump's and transducer's equations (keeping their own annotations, given by [12]):

$$\bar{Q}_p=k_{pn}\bar{n}\,, \tag{11}$$

$$\bar{x}=k_{es}\bar{n}-k_u\bar{u}\,. \tag{12}$$

As far as adjustments of maximum speed are made during ground tests, the term(s) containing $\bar{u}$ can be excluded. Using above determined Eqs. (7) to (12), system's block diagram with transfer functions was built, as depicted in Fig. 4.

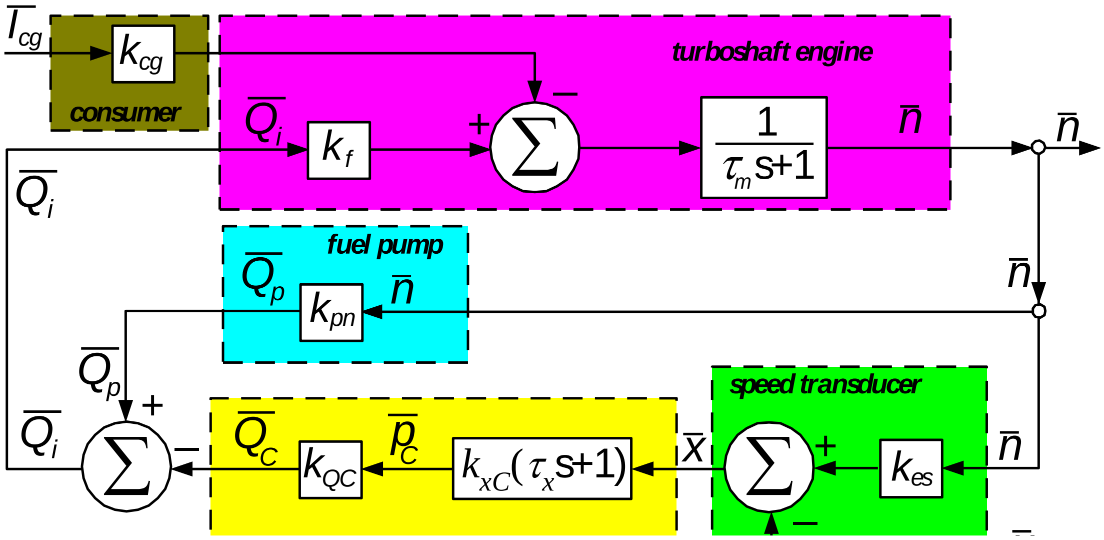

FIG. 4. APU system's block diagram with transfer functions

Furthermore, from Eqs. (7) to (12), a much simpler single equation may be obtained, equation which expresses the dependence $\bar{n}=f\left(\bar{I}_{cg}\right)$ and will give the expression for system's transfer function, as follows:

$$\left[\left(\tau_m-a\tau_x\right)s+1-k_f k_{pn}+a\right]\cdot\bar{n}=-k_{cg}\bar{I}_{cg}\,, \tag{13}$$

equivalent to

$$\bar{n}=-\frac{k_{cg}}{\left(\tau_m-a\tau_x\right)s+1-k_f k_{pn}+a}\bar{I}_{cg}\,, \tag{14}$$

where the term a has the expression $a = k_{QC}k_{xC}k_{es}$. Thus, system's transfer function expression becomes

$$H_n(s) = -\frac{k_{cg}}{(\tau_m - a\tau_x)s + 1 - k_f k_{pn} + a}. \tag{15}$$

When mounted on the ground test facility (see Fig. 5), APU's fuel system operates a little different, because of the secondary discharge pipe cancellation; obviously, the mathematical model form modifies (as presented in [15]). The most important consequence is that the fuel flow rate Q_R, given by Eq. (4), becomes null, so Eq. (6) has a new form, as follows:

$$Q_C - Q_d = S_p \frac{\mathrm{d}x}{\mathrm{d}t} + \beta_f V_C \frac{\mathrm{d}p_C}{\mathrm{d}t}. \tag{16}$$

Furthermore, k_{RC} – coefficient becomes also null, which implies both k_{xC} and a coefficients values modifying:

$$k'_{xC} = \frac{x_0}{(k_{CC} - k_{dC})p_{C0}}; \; a' = k_{QC}k'_{xC}k_{es}. \tag{17}$$

As presented in [15], system's time constant $(\tau_m - a'\tau_x)$ is also affected, becoming smaller; meanwhile, the term $1 - k_f k_{pn} + a'$ becomes bigger, but system transfer function keeps the same form:

$$H_n(s) = -\frac{k_{cg}}{(\tau_m - a'\tau_x)s + 1 - k_f k_{pn} + a'}. \tag{18}$$

5. STUDIES CONCERNING SYSTEM'S STABILITY AND QUALITY

In both of above-presented situations (see Eqs. (15) or (18)) system's transfer function is a first order one. In order to assure system's stability, it is compulsory that its characteristic polynomial coefficients should have the same sign. Consequently, system's stability condition shall be expressed as:

$$(\tau_m - a\tau_x)(1 - k_f k_{pn} + a) > 0. \tag{19}$$

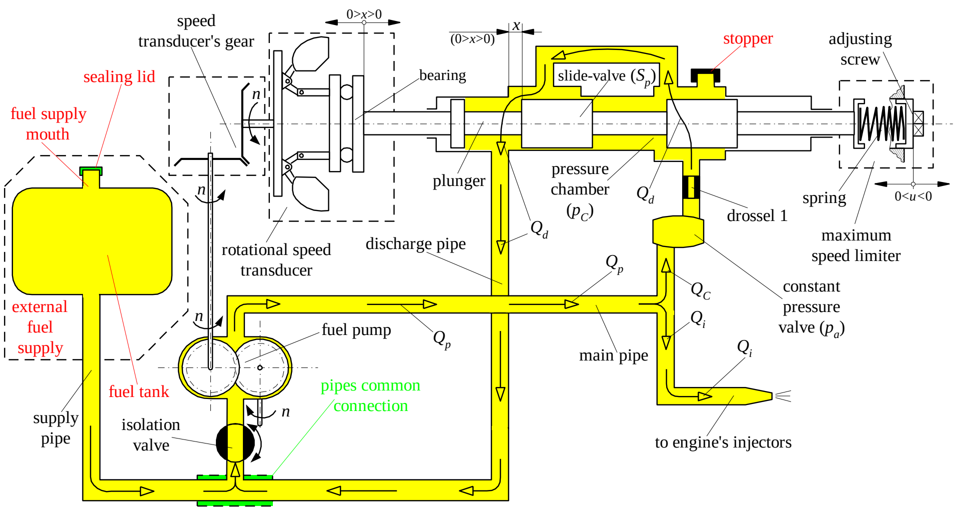

FIG. 5. APU mounted on test facility (fuel system with external supply and different discharges)

Because of its definition formula(s), the term a is strictly positive. Meanwhile, from [11] and [12] it results that the term $1-k_f k_{pn}$ must be strictly positive, in order to assure the stability of the engine-fuel pump connection. Consequently, from Eq. (19) it remains that only the first term $\tau_m - a\tau_x$ has to be discussed, so $\tau_m - a\tau_x > 0$, which gives (considering formulas for a and τ_m):

$$S_p < \frac{k_{dx}}{k_{QC}k_{xC}k_{es}}\tau_m . \tag{20}$$

If $1-k_f k_{pn} < 0$, the engine-fuel pump connection becomes unstable, which means that, in order to keep the same condition (20), a supplementary condition is required. Consequently, for $1-k_f k_{pn} + a > 0$, this supplementary condition has the form:

$$a > \left|1-k_f k_{pn}\right|, \tag{21}$$

otherwise the term $\tau_m - a\tau_x$ should become negative, and the (20)-condition becomes

$$S_p > \frac{k_{dx}}{k_{QC}k_{xC}k_{es}}\tau_m . \tag{22}$$

System's co-efficient $\tau_u = \dfrac{\tau_m - a\tau_x}{k_{cg}}$ (APU's time constant) and $\rho_u = \dfrac{1-k_f k_{pn} + a}{k_{cg}}$ (APU's stability co-efficient) were experimentally determined using the test facility for ground operation $(H=0, V=0)$ and analytically estimated for other flight regimes $(H \neq 0, V \neq 0)$, having as sources mathematical expressions of the involved coefficients $(k_f, k_{pn}, k_{cg}, \ldots)$ and their variations with respect to the flight regime (as presented in [12], [13] and [10]).

The obtained results are graphically presented in Fig. 6, for a range of 10000 m of altitude and 600 km/h of speed; one can observe that τ_u is always positive, but grows for more intense flight regimes; it means that the engine becomes slower in action, its responses at acceleration and/or decelerations becoming longer. The stability coefficient ρ_u has a different behavior, decreasing for more intense flight regimes; it becomes negative at high altitudes and flight speeds (altitudes bigger than 8000 m and speeds bigger than 520 km/h), which leads to an unstable APU behavior.

System's quality was studied for two situations: a) idling engine (without generator's load); b) step input of the electrical load $\bar{I}_{cg}$. Results (step responses of the main output – the engine's speed n, as well as of a secondary output – injection fuel flow rate Q_i) are presented in Fig. 7 and were calculated using Matlab-Simulink simulations, based on system's block diagram with transfer functions (depicted in Fig. 4) and on graph-analytical determined coefficient values (depicted in Fig 6).

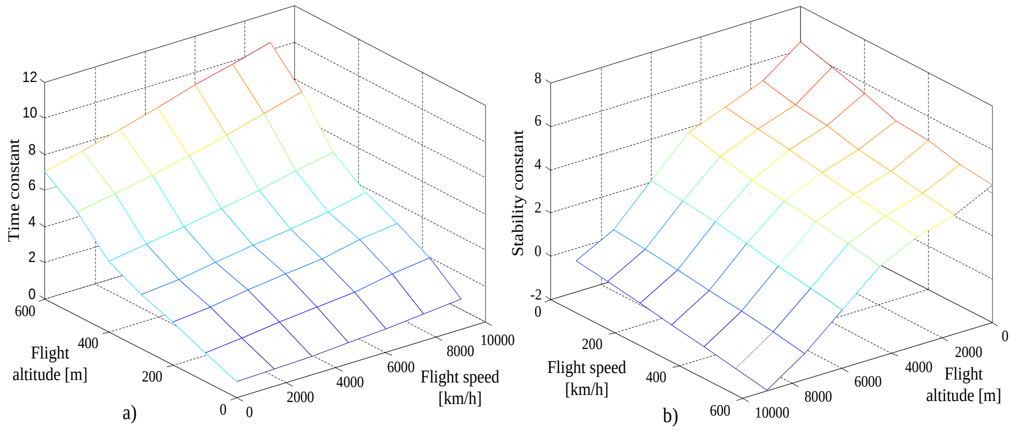

FIG. 6. System's co-efficient values with respect to the flight regime (altitude *H* and flight speed *V*)

Engine speed parameter's behavior is presented in Fig. 7.a), for different flight regimes for both situations: with continuous line – for idling engine, while with dash-dot line for the embedded system. Engine's behavior proves an aperiodic stabile system, with growing static error and stabilization time, the more intense the flight regime is. For the idling engine static errors are positive, while for the embedded engine+generator system static errors are negative and 2…3 times bigger as absolute values. At high altitudes (over 5000 m) the static error, as well as the stabilization time, become prohibitive (because they grow much over the acceptable values), while over 8000 m the system becomes unstable and needs to be assisted by another type of controller.

Figure 7.b) shows the secondary output parameter (fuel flow rate $\overline{Q}_i$) step response, for both above-mentioned situations. Same observations can be made, concerning the aperiodic stability, excepting the fact that static errors for the idling engine is a negative one too.

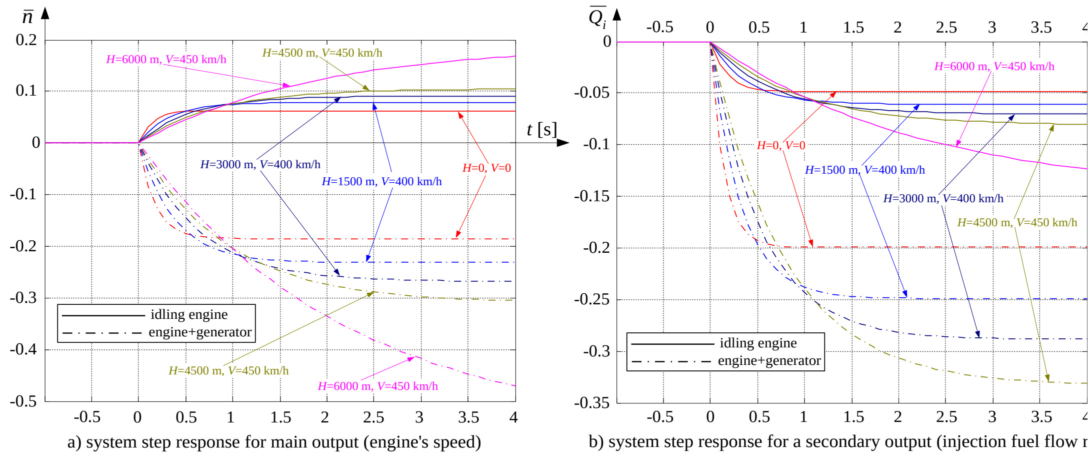

FIG. 7. System's quality: speed and injection fuel flow rate parameters' step response for APU's ground test configuration comparing to APU's airplane configuration

CONCLUSIONS

The paper has studied an APU system as controlled object and has determined his simplified mathematical model, useful for studies. The APU consists of a single shaft turbo-engine, which spins up a dedicated DC electrical generator through a reduction gear. From its non-linear motion equations, using the finite difference method and the Laplace transformation, one has determined the linear non-dimensional mathematical model, as well as its transfer function model, as well as its transfer function.

It has result a first order system (see Eqs. (15) and (18)), its transfer function having a first-degree characteristic polynomial. Consequently, its stability studies were significantly simplified; a simple condition for the stability was obtained, giving information about how to choose the plunger's slide valve frontal surface area S_p with respect to the gas turbine engine time constant τ_m and to the gas turbine engine's fuel system geometry (effective diameters), as well as to flow rate coefficients values.

System's quality studies results, presented in Fig. 7, show stabile aperiodic behavior for both the idling engine and the embedded system engine+generator. However, the system is affected of static errors (positive for idling engine, negative otherwise), especially when the generator supplies external consumers, the bigger the consumers' electrical power are. The system becomes unstable for high altitudes and speeds, so it can be used as it is only for low speed turboprop airplanes, or, with some other fuel/speed controller and/or settings, for high altitude and speed airplanes (such as medium- or long-haul passenger airplanes with turbofan engines).

This study was realized for experimental ground test operation and analytically estimated for several flight regimes (flight speeds and altitudes), but it may be extended for other APUs, using same method and approach.

REFERENCES

[1] Abraham, R. H. *Complex dynamical systems*, Aerial Press, Santa Cruz, California ,1986;

[2] Dinca, L. *On-board Hydro-pneumatic Equipment and Systems*, Universitaria Publisher, Craiova, 2008;

[3] Gorinevsky, D., Dittmar, K., Milaraswamy, D., Nwadiogbu, E., Model-based Diagnostics for an Aircraft Auxiliary Power Unit, *IEEE Conference on Control Applications, Glasgow, Scotland, 18-20 September, 2002*;

[4] Ispas, St., *Turbojet engine*. Tehnica Publisher, Bucuresti, 1984;

[5] Lungu, R., Tudosie, A., Dinca, L. *Fluid Mechanics and Technical Thermodynamics*. SITECH Publisher, Craiova, 2004;

[6] Lungu, R. *Flight Apparatus Automation*. "Universitaria" Publisher, Craiova, 2000;

[7] Mattingly, J. D. *Elements of Gas Turbine Propulsion*. McGraw-Hill, New York, 1996;

[8] Mattingly, J. D., Heiser, W. H., Pratt, D. T. *Aircraft Engine Design*. AIAA Education Series, Reston (VA), 2002;

[9] Rotaru, C., Andres-Mihăilă, M., Matei, P.G. An Extended Combustion Model for the Aircraft Turbojet Engine, *International Journal of Turbo & Jet Engines*, Volume 31, Issue 3, Pp. 229-237, ISSN 0334-0082, DOI 10.1515/tjj-2013-0048, 2014.

[10] Rotaru, C., Sprinţu, I. State Variable Modeling of the Integrated Engine and Aircraft Dymanics, *10th International Conference on Mathematical Problems in Engineering, Aerospace and Sciences* (ICNPAA 2014), Volume 1637, pp. 889-898, DOI 10.1063/1.4904661, 2014.

[11] Rotaru, C., Arghiropol, A., Barbu, C., Boşcoianu, M. Some Aspects Regarding Possible Improvements in the Performances of the Aircraft Engines, *Proc. of the 6th IASME/WSEAS International Conference on Fluid Dynamics and Aerodynamics*, ISBN 978-960-6766-98-5, pp. 196-201, Greece, 2008;

[12] Tudosie, A. N. *Aerospace Propulsion Systems Automation*. Univ. of Craiova Inprint, Craiova, 2005;

[13] Tudosie, A. N. *Aircraft Gas-Turbine Engine's Control Based on the Fuel Injection Control* in Aeronautics and Astronautics, Max Mulder (Ed), INTECH, Rijeka, Croatia, 2011;

[14] Tudosie, A. N. Turboshaft-type APU for Aircraft as Controlled Object, *International Conference Scientific Research And Education In The Air Force - AFASES 2016*,Volume I, pp. 93-100, ISSN 2247-3143, DOI 10.19062/2247-3143.2016.18.1.12, 2016;

[15] Tudosie, A. N. Ground Test Facility For A Turboshaft-tpe APU TG-16M For Passenger Aircraft, *International Conference Scientific Research And Education In The Air Force-AFASES 2016*,Volume I, pp. 85-92, ISSN 2247-3143, DOI: 10.19062/2247-3173.2016.18.1.11, 2016.

[16] Varga, B., Ovari, G., Kavas, L. The Turbine Inlet Temperature And Compressor Pressure Ratio, The Siamese Twins Of The Gas Turbine Engines. *International Conference Scientific Research And Education In The Air Force-AFASES 2016*,Volume I, pp. 393-398, ISSN 2247-3143, DOI: 10.19062/2247-3173.2016.18.1.53, 2016.