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BEST PATTERN OF MULTIPLE LINEAR REGRESSION

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Abstract: *In the economical domain we often analyze the influence of several causal variables on a resulting variable, using a pattern of multiple linear regression. Among the independent factorial variables taken initially into account in the study, we can deduce throughout the process that a part of them have an insignificant statistic influence on the effect variable. The article presents a method of eliminating insignificant variables and determining the best pattern of multiple linear regression.*

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1. INTRODUCTION

The connection between two or among several factorial variables and a resulting variable is called multiple connection, therefore the choice of the factorial variables is very important so that the variation of the resulting variable should be real. Factorial variables exert a greater or smaller influence on the resulting variable, consequently some of the factorial variables are more important and must be taken into account in the study which is made, while for other variables it is proven that they are not so important for the study of the resulting variable variation and must be eliminated. Factorial or causal variables are ordered according to the importance of their actions on the effect phenomenon and one looks for a regression equation which is the best.

A best pattern of regression can be obtained by the retrograde elimination method, which consists of the successive elimination of the factorial variables taken initially into the multiple regression equation until the pattern becomes the best, carefully observing to statistically verify the emergence criterion.

2. STATISTICAL HYPOTHESIS USED FOR THE CHOICE OF VARIABLES WHICH ARE ELIMINATED FROM THE PATTERN

We take the dependent variable Y and k the independent variabiles; there are $X_1, X_2, ..., X_k$ connected by a multiple regression equation :

$$Y = a_0 + a_1 X_1 + ... + a_{j-1} X_{j-1} + a_j X_j + \\ + a_{j+1} X_{j+1} + a_{j+1} X_{j+1} + ... + a_k X_k + \varepsilon ,$$

where the coefficients' matrix of the pattern is $a^T = \left(a_0\ a_1 ... a_j ... a_k\right)$ and the matrix of the parameter estimators of the pattern is $\hat{a}^T = \left(\hat{a}_0\ \hat{a}_1 ... \hat{a}_j ... \hat{a}_k\right)$, estimators obtained through the smaller quadrants method.

We assume that the estimators obtained are unbiased , having a minimal variance and following the normal law.

Variable X is normal $N(m, \sigma^2)$ when the standardized variable $Z = \frac{X - m}{\sigma}$ follows the reduced normal law $N(0,1)$.

The main diagonal of the covariance matrix of the vector a is formed by the estimators variances, the matrix expression being:

$$V = \sigma^2 \cdot (X^T \cdot X)^{-1} = \sigma^2 \cdot S^{-1},$$

where $S^{-1} = (\hat{s}_{ij}^2)_{(k+1)\times(k+1)}$ therefore:

$a_0 \in N\left(\hat{a}_0, \sigma^2 \cdot \hat{s}_{11}^2\right)$, $a_1 \in N\left(\hat{a}_1, \sigma^2 \cdot \hat{s}_{22}^2\right), \ldots,$

$a_k \in N\left(\hat{a}_k, \sigma^2 \cdot \hat{s}_{(k+1),(k+1)}^2\right).$

If σ^2 is unknown then the variables:

$$Z_j = \frac{a_j - \hat{a}_j}{\sigma \cdot \sqrt{\hat{s}_{j+1,j+1}^2}}, j = \overline{0,k} \qquad (1)$$

Follow the reduced normal law $N(0,1)$.

As σ^2 is unknown this is replaced by the unbiased estimator:

$$s_\varepsilon^2 = \frac{1}{n-k-1} \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i)^2,$$

n is the number of observations, from which we obtain:

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (n-k-1) \cdot s_\varepsilon^2 \qquad (2)$$

The values of the residual variable $\varepsilon_i = y_i - \hat{y}_i$, $\forall i = \overline{1,n}$ are normally distributed, that is $\varepsilon_i \in N(0, \sigma^2), \forall i = \overline{1,n}$ which leads to the conclusion that $\frac{\varepsilon_i}{\sigma} \in N(0,1), \forall i = \overline{1,n}$ and

$$\frac{1}{\sigma^2} \sum_{i=1}^{n} \varepsilon_i^2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \chi_{n-k-1}^2$$

From which we obtain:

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sigma^2 \cdot \chi_{n-k-1}^2 \qquad (3)$$

From (2) and (3) we obtain:

$$s_\varepsilon^2 = \frac{\sigma^2}{n-k-1} \cdot \chi_{n-k-1}^2 \qquad (4)$$

We calculate the estimator average s_ε^2:

$$M(s_\varepsilon^2) = M\left(\frac{\sigma^2}{n-k-1} \cdot \chi_{n-k-1}^2\right) = \\ = \frac{\sigma^2}{n-k-1} M\left(\chi_{n-k-1}^2\right) = \\ \frac{\sigma^2}{n-k-1} \cdot (n-k-1) = \sigma^2 .$$

That is the estimator s_ε^2 id unbiased.

The variables $t_j = \frac{Z_j}{\sqrt{\frac{\chi_{n-k-1}^2}{n-k-1}}}, \forall j = \overline{0,k}$ follow the law Student with $n-k-1$ degrees of freedom, therefore using the relations (1) and (4) we obtain:

$$t_j = \frac{\frac{a_j - \hat{a}_j}{\sigma \cdot \sqrt{\hat{s}_{j+1,j+1}^2}}}{\sqrt{\frac{\chi_{n-k-1}^2}{n-k-1}}} = \frac{\frac{a_j - \hat{a}_j}{\sigma \cdot \sqrt{\hat{s}_{j+1,j+1}^2}}}{\frac{s_\varepsilon}{\sigma}} = \\ = \frac{a_j - \hat{a}_j}{s_\varepsilon \cdot \sqrt{\hat{s}_{j+1,j+1}^2}}, \forall j = \overline{0,k}$$

For a determined value $\hat{a}_{j0}$ and statistical $t_{j,\text{calculat}} = \frac{a_j - \hat{a}_{j0}}{s_\varepsilon \cdot \sqrt{\hat{s}_{j+1,j+1}^2}}$ we set the hypothesis:

$H_0^{(j)} : \hat{a}_j = \hat{a}_{j0}$

$H_1^{(j)} : \hat{a}_j \neq \hat{a}_{j0}$

And if $\left|t_{j,\text{calculat}}\right| > t_{1-\frac{\alpha}{2};n-k-1}$ then we reject the hypothesis $H_0^{(j)}$ and accept the hypothesis $H_1^{(j)}, \forall j = \overline{0,k}$.

For $\hat{a}_{j0} = 0$ we obtain

$$t_{j,\text{calculat}} = \frac{a_j}{s_\varepsilon \cdot \sqrt{\hat{s}_{j+1,j+1}^2}} = \frac{a_j}{s(a_j)}, \forall j = \overline{0,k},$$

these are distributed with Student with $n-k-1$ degrees of freedom, the statistical hypotheses being:

$$\begin{matrix} H_0^{(j)} : \hat{a}_j = 0 \\ H_1^{(j)} : \hat{a}_j \neq 0 \end{matrix} \quad \forall j = \overline{0,k}$$

And for $\left|t_{j,\text{calculat}}\right| > t_{1-\frac{\alpha}{2};n-k-1}$ we reject the hypothesis $H_0^{(j)}$.

The distribution

$F_{\alpha;1,n-k-1}$

is the distribution of the statistics $\left(t_{j,\text{calculat}}\right)^2 = \frac{a_j^2}{s^2(a_j)}$, $\forall j = \overline{0,k}$ and for $\left(t_{j,\text{calculat}}\right)^2 > F_{\alpha;1,n-k-1}$ we reject the null hypothesis $H_0^{(j)} : \hat{a}_j = 0$.

3. RETROGRADE ELIMINATION METHOD TO OBTAIN THE BEST REGRESSION WE DO THE FOLLOWING

3.1. We obtain $\hat{y}_i^L$, $\forall i = \overline{1,n}$ by the smaller quadrants method using all the initial factorial variables $X_1, X_2, ..., X_k$.

3.2. The statistics of the test is:

$$F_{X_j,\text{calculat}} = \left(t_{j,\text{calculat}}\right)^2 = \frac{a_j^2}{s^2(a_j)}, \forall j = \overline{1,k}$$

And we determine $\min_{1\le j\le k}\{F_{X_j,\text{calculat}}\}$ and assume that the searched minimal is $F_{X_r,\text{calculat}}$ or we use the statistics $t_{j,\text{calculat}} = \frac{a_j}{s(a_j)}$, $\forall j = \overline{1,k}$ and where there is r so that $\left|t_{r\ \text{calculat}}\right| = \min_{1\le j\le k}\left\{\left|t_{j\ \text{calculat}}\right|\right\}$.

3.3. We set the hypotheses :

$$H_0^{(r)} : a_r = 0$$
$$H_1^{(r)} : a_r \neq 0,$$

And if $F_{X_r,\text{calculat}} < F_{\alpha;1,n-k-1}$ then we accept the hypothesis $H_0^{(r)}$ therefore the factorial variable X_r is eliminated from the pattern, we write the new fitting equation without X_r and we obtain a partial best regression pattern or, if

$\left|t_{r\ \text{calculat}}\right| < t_{1-\frac{\alpha}{2};n-k-1}$

we accept the hypothesis $H_0^{(r)}$ that is $a_k = 0$ and we obtain the partial best pattern of the stage.

3.4. To the pattern obtained at 3.2 we apply the stages 3.2 and 3.3again until the stage where the obtained result does not allow the elimination of other variables and that final pattern obtained is the best .

Example:

Table 1

Nr. crt.	x_{1i}	x_{2i}	x_{3i}	y_i	x_{1i}^2	x_{2i}^2	x_{3i}^2
1	0,1	3,25	22,3	17,2	0,01	10,5625	412,09
2	0,2	2,90	18,6	22,5	0,04	8,41	345,96
3	0,1	3	21,4	18	0,01	9	457,96
4	0,15	2,8	23,5	20,4	0,0225	7,84	552,25
5	0,3	3,4	25	24,3	0,09	11,56	625
Total	0,85	15,35	108,8	102,4	0,1725	47,3725	2393,26

$$\hat{Y}_i = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + \varepsilon$$

$$\begin{cases} 5a_0 + a_1\sum x_{1i} + a_2\sum x_{2i} + a_3\sum x_{3i} = \sum y_i \\ a_0\sum x_{1i} + a_1\sum x_{1i}^2 + a_2\sum x_{1i}x_{2i} + a_3\sum x_{1i}x_{3i} = \sum x_{1i}y_i \\ a_0\sum x_{2i} + a_1\sum x_{1i}x_{2i} + a_2\sum x_{2i}^2 + a_3\sum x_{2i}x_{3i} = \sum x_{2i}y_i \\ a_0\sum x_{3i} + a_1\sum x_{1i}x_{3i} + a_2\sum x_{2i}x_{3i} + a_3\sum x_{3i}^2 = \sum x_{3i}y_i \end{cases} \quad (5)$$

Table 1. Follow up

$x_{1i}x_{2i}$	$x_{1i}x_{3i}$	$x_{2i}x_{3i}$	$x_{1i}y_i$	$x_{2i}y_i$	$x_{3i}y_i$	y_i^2
0,325	2,03	65,975	1,72	55,9	349,16	295,84
0,58	3,72	53,94	4,4	65,25	418,5	506,25

0,3	2,14	64,2	1,8	54	385,2	324
0,42	3,525	65,8	3,06	57,12	479,4	416,16
1,02	7,5	85	7,29	82,62	607,5	590,49
2,645	18,915	334,915	18,37	314,89	2239,76	2132,74

$$\begin{cases} 5a_0 + 0{,}85a_1 + 15{,}35a_2 + 108{,}8a_3 = 102{,}4 \\ 0{,}85a_0 + 0{,}1725a_1 + 2{,}645a_2 + 18{,}915a_3 = 18{,}37 \\ 15{,}35a_0 + 2{,}645a_1 + 47{,}3725a_2 + 334{,}915a_3 = 314{,}89 \\ 108{,}8a_0 + 18{,}915a_1 + 334{,}915a_2 + 2393{,}26a_3 = 2239{,}76 \end{cases} \quad (6)$$

$$X^T \cdot X = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0{,}1 & 0{,}2 & 0{,}1 & 0{,}15 & 0{,}3 \\ 3{,}25 & 2{,}9 & 3 & 2{,}8 & 3{,}4 \\ 20{,}3 & 18{,}6 & 21{,}4 & 23{,}5 & 25 \end{pmatrix} \times$$

$$\times \begin{pmatrix} 1 & 0{,}1 & 3{,}25 & 20{,}3 \\ 1 & 0{,}2 & 2{,}9 & 18{,}6 \\ 1 & 0{,}1 & 3 & 21{,}4 \\ 1 & 0{,}15 & 2{,}8 & 23{,}5 \\ 1 & 0{,}3 & 3{,}4 & 25 \end{pmatrix} = S$$

$$S = X^T \cdot X = \begin{pmatrix} 5 & 0{,}85 & 15{,}35 & 108{,}8 \\ 0{,}85 & 0{,}1725 & 2{,}645 & 18{,}915 \\ 15{,}35 & 2{,}645 & 47{,}3725 & 334{,}915 \\ 108{,}8 & 18{,}915 & 334{,}915 & 2393{,}26 \end{pmatrix}$$

$$\det S = 5^4 \begin{vmatrix} 1 & 0{,}17 & 3{,}07 & 21{,}76 \\ 0{,}17 & 0{,}0345 & 0{,}529 & 3{,}783 \\ 3{,}07 & 0{,}529 & 9{,}4745 & 66{,}983 \\ 21{,}76 & 3{,}783 & 66{,}983 & 478{,}652 \end{vmatrix} = 0{,}535286$$

$$S^{-1} = \begin{pmatrix} 51{,}843741 & 21{,}202339 & -12{,}711121 & -0{,}745634 \\ 21{,}202339 & 52{,}152074 & -5{,}027453 & -0{,}672514 \\ -12{,}711121 & -5{,}027453 & 5{,}100591 & 0{,}096187 \\ -0{,}745634 & -0{,}672514 & -0{,}096187 & 0{,}053091 \end{pmatrix}$$

System (2) written metrical $S \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

to the left with S^{-1} it becomes:

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 51{,}843741 & 21{,}202339 & -12{,}711121 & -0{,}745634 \\ 21{,}202339 & 52{,}152074 & -5{,}027453 & -0{,}672514 \\ -12{,}711121 & -5{,}027453 & 5{,}100591 & 0{,}096187 \\ -0{,}745634 & -0{,}672514 & -0{,}096187 & 0{,}053091 \end{pmatrix} \times \begin{pmatrix} 102{,}4 \\ 18{,}37 \\ 314{,}89 \\ 2239{,}76 \end{pmatrix}$$

From which

$a_0 = 25{,}6399463$

$a_1 = 39{,}78848117$

$a_2 = -3{,}28379714$

$a_3 = -0{,}08423005$

Therefore

$$Y = 25{,}6399463 + 39{,}78848117\ X_1 - 3{,}28379714\ X_2 - 0{,}08423005\ X_3$$

represents the multiple linear regression pattern obtained after the fitting using all the factorial variables.

We determine the statistics $t_{X_j \text{calculat}}$, $j = 1,2,3$ so that :

$$t_{X_j \text{calculat}} = \frac{a_j}{s(a_j)} \quad \text{where}$$

$$s(a_j) = s_\varepsilon \cdot \hat{s}_{j+1,j+1}, \ j = 1,2,3$$

$$s_\varepsilon^2 = \frac{1}{5-3-1}\sum_{i=1}^{5}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{5}(y_i - a_0 - a_1x_{1i} - a_2x_{2i} - a_3x_{3i})^2 =$$

$$= \sum_{i=1}^{5}(y_i - 25{,}6399463 - 39{,}78848117x_{1i} + 3{,}28379714x_{2i} + 0{,}08423005x_{3i})^2 = 0{,}003906$$

$$t_{X_1 \text{calculat}} = \frac{a_1}{s(a_1)} = \frac{a_1}{s_\varepsilon \hat{s}_{22}} = \frac{39{,}78848117}{\sqrt{0{,}003906 \cdot 52{,}152074}} = \frac{39{,}78848117}{0{,}451338} = 88{,}1811$$

$$t_{X_2\text{calculat}}=\frac{a_2}{s(a_2)}=\frac{a_2}{s_\varepsilon \hat{s}_{33}}=$$
$$=\frac{-3{,}28379714}{\sqrt{0{,}003906\cdot 5{,}100591}}=$$
$$=\frac{-3{,}28379714}{0{,}1411485}=-23{,}2648$$

$$t_{X_3\text{calculat}}=\frac{a_3}{s(a_3)}=\frac{a_3}{s_\varepsilon \hat{s}_{44}}=$$
$$=\frac{-0{,}08423005}{\sqrt{0{,}003906\cdot 0{,}053091}}=$$
$$=\frac{-0{,}08423005}{0{,}01440047}=-5{,}8491$$

$$\min_{1\le j\le 3}\left\{\left|t_{X_j\text{calculat}}\right|\right\}=$$
$$=\min\{88{,}1811;23{,}2648;5{,}8491\}=$$
$$=5{,}8491=\left|t_{X_3\text{calculat}}\right|$$

The hypothesis $H_0^{(3)}: a_3=0$ is accepted if $\left|t_{X_3\text{calculat}}\right|<t_{1-\frac{\alpha}{2},5-3-1}$.

We consider

$$\alpha=0{,}05\Rightarrow t_{1-\frac{\alpha}{2};1}=t_{0{,}975;1}=12{,}706$$

and indeed

$$\left|t_{X_3\text{calculat}}\right|=5{,}8491<12{,}706=t_{1-\frac{\alpha}{2},5-3-1}$$

So we eliminate the 3rd column from the matrix X and we obtain :

$$X=\begin{pmatrix}1 & 0{,}1 & 3{,}25\\ 1 & 0{,}2 & 2{,}9\\ 1 & 0{,}1 & 3\\ 1 & 0{,}15 & 2{,}8\\ 1 & 0{,}3 & 3{,}4\end{pmatrix}$$ then we determine the

matrixes $S=X^T\cdot X$ and $B=X^T\cdot Y$

$$S=X^T\cdot X=\begin{pmatrix}1 & 1 & 1 & 1 & 1\\ 0{,}1 & 0{,}2 & 0{,}1 & 0{,}15 & 0{,}3\\ 3{,}25 & 2{,}9 & 3 & 2{,}8 & 3{,}8\end{pmatrix}\times$$
$$\times\begin{pmatrix}1 & 0{,}1 & 3{,}25\\ 1 & 0{,}2 & 2{,}9\\ 1 & 0{,}1 & 3\\ 1 & 0{,}15 & 2{,}8\\ 1 & 0{,}3 & 3{,}4\end{pmatrix}=$$
$$=\begin{pmatrix}5 & 0{,}85 & 15{,}35\\ 0{,}85 & 0{,}1725 & 2{,}645\\ 15{,}35 & 2{,}645 & 47{,}3725\end{pmatrix}$$

$$B=X^T\cdot Y=\begin{pmatrix}1 & 1 & 1 & 1 & 1\\ 0{,}1 & 0{,}2 & 0{,}1 & 0{,}15 & 0{,}3\\ 3{,}25 & 2{,}9 & 3 & 2{,}8 & 3{,}8\end{pmatrix}\times$$
$$\times\begin{pmatrix}17{,}2\\ 22{,}5\\ 18\\ 20{,}4\\ 24{,}3\end{pmatrix}=\begin{pmatrix}102{,}4\\ 18{,}37\\ 314{,}89\end{pmatrix},$$

The adequate extended matrix is $A\left(S\middle|B\middle|I_3\right)$, that is

$$A=\begin{pmatrix}5 & 0{,}85 & 15{,}35 & 102{,}4 & 1 & 0 & 0\\ 0{,}85 & 0{,}1725 & 2{,}645 & 18{,}37 & 0 & 1 & 0\\ 15{,}35 & 2{,}645 & 47{,}3725 & 314{,}89 & 0 & 0 & 1\end{pmatrix}$$

We apply Gauss method and obtain :

$$A'\left(I_3\middle|B'\middle|S^{-1}\right)=$$
$$=\begin{pmatrix}1 & 0 & 0 & \frac{111174{,}4}{4547} & \frac{188117}{4547} & \frac{53460}{4547} & -\frac{63940}{4547}\\ 0 & 1 & 0 & \frac{176036}{4547} & \frac{53460}{4547} & \frac{198400}{4547} & -\frac{28400}{4547}\\ 0 & 0 & 1 & -\frac{15628}{4547} & -\frac{63940}{4547} & -\frac{28400}{4547} & \frac{22400}{4547}\end{pmatrix}$$
$$=\begin{pmatrix}1 & 0 & 0 & 24{,}450055 & 41{,}371674 & 11{,}757203 & -14{,}062019\\ 0 & 1 & 0 & 38{,}714757 & 11{,}757203 & 43{,}633165 & -6{,}245876\\ 0 & 0 & 1 & -3{,}436991 & -14{,}062019 & -6{,}245876 & 4{,}926325\end{pmatrix}$$

After this stage the partial best regression pattern is:

$$Y=24{,}450055+38{,}714757X_1-3{,}436991X_2$$

and $s_\varepsilon^2 = \frac{1}{5-2-1} \cdot \sum_{i=1}^{5} (y_i - 24{,}450055 -$

$-38{,}714757 x_{1i} + 3{,}436991 x_{2i})^2 =$

$= \frac{1}{2} \cdot 0{,}138513298 = 0{,}069256649$

The calculated values of the test ***t*** are:

$$t_{X_1 \text{calculat}} = \frac{38{,}714757}{\sqrt{0{,}069256649 \cdot 43{,}633165}} =$$

$$= \frac{38{,}714757}{1{,}738357} = 22{,}2709$$

$$t_{X_2 \text{calculat}} = \frac{-3{,}436991}{\sqrt{0{,}069256649 \cdot 4{,}926325}} =$$

$$= -\frac{3{,}436991}{0{,}584107} = -5{,}8842$$

and $\min\{|t_{X_1 \text{calculat}}|, |t_{X_2 \text{calculat}}|\} = |t_{X_2 \text{calculat}}| = 5{,}8842$.

For

$\alpha = 0{,}01 \quad t_{1-\frac{\alpha}{2};5-2-1} =$

$= 9{,}925 > |t_{X_2 \text{calculat}}| = 5{,}8842$

resulted which requires the acceptance of the hypothesis $H_0^{(2)} : a_2 = 0$, therefore we eliminate from the pattern the variable X_2 and the equation of the best regression pattern after this stage is:

$$Y = 24{,}450055 + 38{,}714757 X_1$$

for which

$$s_\varepsilon^2 = \frac{1}{5-1-1} \sum_{i=1}^{5} (y_i - 24{,}450055 -$$

$$-38{,}714757 x_{1i})^2 =$$

$$= 186{,}581866$$

the adequate extended matrix is:

$$A = \begin{pmatrix} 5 & 0{,}85 & 102{,}4 & 1 & 0 \\ 0{,}85 & 0{,}1725 & 18{,}37 & 0 & 1 \end{pmatrix} =$$

$$= (S|C|I_3), C = X^T \cdot Y$$

And $A' = \left(I_3 | C' | S^{-1} \right) =$

$$= \begin{pmatrix} 1 & 0 & 14{,}639286 & 1{,}232143 & -6{,}071429 \\ 0 & 1 & 34{,}357143 & -6{,}071429 & 35{,}714286 \end{pmatrix}$$

And

$$t_{X_1 \text{calculat}} = \frac{38{,}714757}{\sqrt{186{,}581866 \cdot 35{,}714286}} =$$

$$= \frac{38{,}714757}{81{,}631110} = 0{,}4743$$

For

$\alpha = 0{,}05 \ t_{X_1 \text{calculat}} = 0{,}4743 > t_{1-\frac{\alpha}{2};3} = 3{,}482$

therefore we reject the hypothesis $H_0^{(1)} : a_1 = 0$, so the best pattern is $Y = 24{,}450055 + 38{,}714757 X_1$.

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